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Rough Work

1. Find the fraction of each quantity? The ratio of boys to girls is 2:3. The total number of students is 30. Find the number of boys and girls. What fraction of students are boys and what fraction are girls? **Theory:** Ratio is used to compare two or more quantities. We can also use ratios to find each quantity as a fraction of the total. For example in the above problem, the ratio of boys to girls is 2:3.

   Boys: Girls = 2:3

   So 2 parts are boys and 3 parts are girls and so total parts becomes 5 (2 + 3).

   So fraction of boys is $\frac{2}{5}$ and fraction of girls is $\frac{3}{5}$.

2. Angles of a quadrilateral and check it is cyclic?

Type 0: Introduction

In our everyday life, we compare two things of the same type.

For example, if the height of Alex is 150 cm and that of Mike is 140 cm, then we can compare their height and say Alex is 10 cm (150-140=10cm) taller than Mike.

Similarly if Alex has 5 pencils and if Mike has 9 pencils, then we say Mike has 4 pencils (9-5=4) extra than Alex.

In the above cases, we made a comparison using the difference (subtraction). However, difference (subtraction) is not always helpful.

Consider the case of comparing the length of a train with a car. If we subtract the length of the car from the train's length, it does not actually tell us how long the train is compared to the car.

A better comparison could be how many cars can be arranged in a line so as to match the length of the train.

So if a train is say 80m in length and car is 5m in length,

80 ÷ 5 = 40 tells that train is 40 times larger than a car.

This is known as comparison by Division. The comparison by division is known as "Ratio".

Example: The number of students opting for Maths is 40 and those opting Science is 8 in an Olympiad exam. Then number of students in Maths is 40 ÷ 8 = 5 times the number of students in Maths.

We express this as "Ratio of number of students opting Maths to Science" = $\frac{40}{8} = \frac{5}{1} = 5 : 1$

So 40:8 = 5:1

Ratios can also be expressed as fractions. The first number in a ratio (a:b) becomes the Numerator and the second number becomes the Denominator.

So a:b = $\frac{a}{b}$

So using the above example,

40:8 = $\frac{40}{8} = \frac{5}{1} = 5 : 1$

Two quantities can be compared only if they are in same units.

If the length of a car is say 6 meters and the length of a toy car is say 2 cm

then the ratio of their lengths would be

$$\frac{Car's\ length}{Toy\ car's\ length} = \frac{6}{2} = \frac{3}{1} = 3 : 1$$

It means the car's length is three times the toy car's length. But this is not true.

So what is wrong? We see that the car's length is in meters but toy car's is in centimeters.

So a proper comparison would be

$$\frac{Car's\ length}{Toy\ car's\ length} = \frac{6m}{2m} = \frac{600cm}{2cm} = \frac{300}{1}$$

So the car is 300 times the length of a toy car.

Problem: Roger and Rafa started a business and invested money in the ratio 1:2. After one year they got a profit of $2100.

Roger said "Let's divide the money equally", but Rafa said "I should get more as I have invested more".

So to divide the profit in the ratio of money invested.

Ratio = 1:2

Sum of terms of the ratio = 1+2=3

It means if profit is $3, then Roger would get 1 and Rafa would get 2

So Roger would get 1 part out of 3 = $\frac{1}{3}$

and Rafa would get 2 part out of 3 = $\frac{2}{3}$

Total profit = $2100

So Roger's share = $\frac{1}{3}$ x 2100 = $700

Rafa's share = $\frac{2}{3}$ x 2100 = $1400

Problem: There are 50 students in a class. If the number of boys is 20 and rest are girls, find the ratio of:

a) Number of boys to number of girls

b) Number of girls to number of boys

Solution: Total Students = 50

Boys = 20

Girls = 50-20=30

a) Ratio of number of boys to number of girls

$= 20:30 = 2:3 = \frac{2}{3}$

b) Ratio of number of girls to number of boys

$= 30:20 = 3:2 = \frac{3}{2}$

Problem: Give two equivalent ratios of 9:6?

Solution: Ratio $9:6 = \frac{9}{6} = \frac{3}{2} = 3:2$.

Here 3:2 is equivalent of 9:6

Also,

$9:6 = \frac{9}{6} = \frac{9*2}{6*2} = \frac{18}{12} = 18:12$

So 18:12 is also equivalent of 9:6.

We can get equivalent ratios by multiplying or dividing the Numerator & Denominator by the same number.

Problem: Divide $80 in the ratio 2:3?

Solution: Ratio = 2:3

Sum of terms = 2+3 = 5.

So the first is 2 out of 5 (or $\frac{2}{5}$)

Second term is 3 out of 5 (or $\frac{3}{5}$)

Now dividing $80 in the ratio 2:3

1st term = $\frac{2}{5} * 80 = 2 * 16 = 32$

2nd term = $\frac{3}{5} * 80 = 3 * 16 = 48$

So, dividing $80 in the ratio 2:3 is 32 & 48.Check (32+48 = 80)

Problem: Fill in the missing numbers:-

$\frac{14}{21} = \frac{\ }{3} = \frac{6}{\ }$

Solution: We see $\frac{14}{21} = \frac{2}{3}$ (equivalent fraction).

So the first missing number is 2.

Now $\frac{2}{3} = \frac{6}{9} = \frac{2*3}{3*3}$

So the second missing number is 9.

Problem: Ratio of distance of Amanda's house to school and Jessica's house to school is 2:1.

a. Who lives nearer to school?

b. Complete the table which shows some possible distances of Amanda's and Jessica's home from school.

Distance of Amanda’s house to school	20	?	6	?
Distance of Jessica’s house to school	10	6	?	1

c. If the ratio of distance of Amanda's house to school to the distance of Madison's house to school is 1:2, then who lives nearer to school?

Solution: a. Ratio of distance of school from Amanada's house to school from Jessica's house = 2:1

It means Amanda's house is twice the distance than Jessica's house.

So Jessica lives closer to the school.

So we have:

Distance of Amanda's house to school	20	12	6	2
Distance of Jessica's house to school	10	6	3	1

c) Ratio of Amanda & Madison's distance to school

1:3

So Amanda lives closer to school. And Madison must be twice distant as compared to Amanda.

Problem:- Mira earns $20,000 and saves $5000. Find the ratio of:-

a) Money earned to money saved.

b) Money saved to money spent.

Solution: Total earning = $20,000

Savings = $5000

So money spent = $20,000 - 5000

= $15000

a) Ratio of money earned to money saved

= 20,000 : 5000

= 4:1

b) Ratio of money saved to money spent

= 5000 : 15000

= 1:3

Problem:- In a school, out of 3000 students, 1200 are boys. Find the ratio of:

(a) Number of boys to total number of students

(b) Number of girls to number of boys

(c) Number of girls to total number of students.

Solution:- Total Students = 3000

Boys = 1200

Girls = 3000-1200 = 1800.

(a) Ratio of number of boys to total number of students

= 1200 : 3000 = $\frac{1200}{3000} = \frac{2}{5}$

(b) Ratio of number of girls to number of boys = 1800 : 3000 = $\frac{1800}{3000} = \frac{3}{5}$ = 3 : 5 **Problem:-** The cost of a 10 pen is Rs 80 and the cost of a dozen pencil is Rs 48. Find the ratio of the cost of a simple pen to the cost of a simple pencil?

Solution:- Cost of 10 pen = RS 80

= cost of 1 pen = 80 ÷10 = Rs 8

Note : 1 dozen means 12.

Now Cost of 12 pencils = 48

=) Cost of 1 pencil = 48 ÷ 12 = Rs 4.

Now Ratio of cost of a pen to cost of a pencil

8:4 = 2:1.

Problem: A father wants to divide $30 between his daughter Jasmine & Mirra in the ratio of their age. If Jasmine is 12 years and Mirra is 8 years old, find how much money did each get?

Solution: Jasmine's age = 12 years

Mirra's age = 8 years.

So Ratio of age of Jasmine to Mirra

$= 12:8 = \frac{12}{8} = \frac{3}{2} = 3:2$

So Ratio of age = 3:2. So the daughters will get the $30 in this ratio.

Sum of the terms of ratio = 3+2 = 5

So it means if there is $5,

then Jasmine will get $3 (or $\frac{3}{5}$)

and Mirra will get $2 (or $\frac{2}{5}$ of 5).

Here the total money = $30

So Jasmine will get = $\frac{3}{5}$*30 = $18

Mirra will get = $\frac{2}{5}$*30 = $12

So Jasmine will get $18 and Mira will get $12. (18+12=30)

Problem:In a box containing 12 dozen biscuit packets, 30 were found to be cracked.

a. Calculate the ratio of cracked to uncracked biscuits packets, Express the ratio in simplest form and in the lon form?

b. If this ratio is the same for all packets, calculate the number of cracked biscuit packets in 80 dozen packetsc.

c. Calculate the total number of biscuit packets to be bought so that 285 packets are uncracked?

Solution: Total packets of biscuit = 12 dozens

Now 1 dozen = 12 units

So 12 dozen means: 12 × 12 = 144 biscuit packets

Out of 144 packets, 30 are cracked.

So number of uncracked packets = 144 - 30 = 114

(A) Ratio of cracked to uncracked packets of biscuit = 30:114 = 5 : 19 (divide by 6)

And Ratio of cracked to total number of packets = 30:144 = 5:24 (Divide by 6)

So out of every 24 packets, 5 are cracked. Or $\frac{5}{24}$ of total number of packets would be cracked.

In the form of 1:n, we have to make the first term of the ratio equal to one.So ratio is 5:24So dividing both terms by 5, we get1:4.8.

(B) Number of packets in 8 dozen = 12 x 8 = 96

Now if $\frac{5}{24}$ of packets are cracked. It means 5 packets are cracked out of every 24 packets. So the number of cracked packets out of 60 packets = $\frac{5}{24} * 96$ = 20

(C) We have 114 uncracked packets out of 144 total packets.

We have 144 uncracked packets out of 144 total packets.

So 1444 total packets give 114 good (uncracked) packets.

Now we want 285 good (uncracked) packets. So we want the find the total number of packets needed.

So we use the proportions $\frac{114}{144} = \frac{285}{x}$

where x is the total number of packets.Using cross multiplication, we get 114*x = 285*144

X = $\frac{285*144}{114}$ = 360

So a total of 360 packets (30 dozens) are needed to get 285 uncracked biscuit packets.

Type 1: Ratio

What is a Ratio? A ratio is a way to compare two quantities.

For example if we have 5 pencils and 3 erasers then we can compare pencils and erasers as:-

The ratio of pencils to erasers is 5:3.

The symbol : (colon) is read as "is to"

So 5:3 is to be read as "5 is to 3".

Different ways of writing ratios:-

Ratio can be written as fractions

① 5:3 (5 is to 3) can be written as $\frac{3}{5}$

② Also 5:3 = $\frac{5}{3}$ and 3:5 = $\frac{3}{5}$

So order matters in ratio

③ We can simplify ratio as we simplify fractions.

For example:- The number of boys in class 6 is 10 and number of girls is 14.

So the ratio of number of boys to girls

= 10:14 (10 is to 14).

$= \frac{10}{14} = \frac{5}{7}$

It means for every 5 boys, there are 7 girls.

Equivalent Ratios:- So we can say 10:14 is equivalent to 5:7.

Or 10:14 = 5:7

Similarly 6:9 = 2:3

20:50 = 2:5 and so on

Problem: The number of students playing football in class 6 is 10 and in class 7 is 15. Express this in the form of ratio. Simplify the ratio obtained.

Solution: Number of class 6 students playing football = 10

Number of class 7 students playing football = 15

So ratio = 10:15 = $\frac{10}{15}$

Simplifying, we get $\frac{10}{15} = \frac{2}{3}$

Note:- when writing a ratio in its simplest form, the Numerator & the Denominator must always be whole numbers. They cannot be fractions or decimals.

Problem: A coaching academy has 10 coaches and 120 players. Calculate the coach: player in the simplest form.

Solution: Number of coaches = 10

Number of players = 120

So Ratio = 10:120 = $\frac{10}{120} = \frac{1}{12}$ = 1:12

It means there are 12 players for every coach.

Problem:- The ratio of voltage to current in an electric circuit is 2:1. If the current in the circuit is 10 Ampere find the value of voltage ?

Solution:- Ratio of Voltage to current = 2:1

=> Voltage : Current = 2 : 1 = $\frac{2}{1}$

Now if current = 10 Amperes, then to calculate voltage, we multiply

both the terms of ratio by 10

So 2:1 = 2x10: 1x10

= 20:10

So if current is 10 Ampere, the voltage would be 20 Volts

Another method:- Voltage: Current = 2:1

It means Voltage is twice of current.

So if current is 10 Amp, then voltage would be 20 Volts.

Sometimes ratios can be written in the form 1:n.

If we are given any ratio, it can be converted to form 1:n.

It means we make the first number 1. And calculate the value of 2nd number

For example: (a) If ratio is 3:12, then we first make the first number 1 by dividing by 3. We do the same with the second number. So

$3:12 = \frac{3}{3} : \frac{12}{3} = 1 : 4$

(b) Express 4:22 in the form 1:n

we divide both the numbers of the ratio 4:22 by 4 to get

$\frac{4}{22} = \frac{4}{4} : \frac{22}{4} = 1 : 55$

It means for every 1 of first quantity,there are 5.5 of second quantity.

Problem: A paint called "Ocean Blue" is created by mixing Blue & white in the ratio 3:4.

A different shade "Volcanic Blue" is created using Blue and white paint in the ratio 5:6.

Calculate which mixture is darker assuming blue colour adds to darkness.

Solution: "Forest Blue" contains Blue and white in the ratio

Blue : white = 3:4

"Volcanic Blue" contains Blue & white in the ratio

Blue : white = 5:6

Now we compare the two ratios by making the Blue part same in both ratios

Forest Blue = 3:4 = 15:20 (x by 5)

Volcanic Blue = 5:6 = 15:18 (x by 3)

So in Forest Blue, 15 parts of Blue has 20 parts of

For the same amount of Blue (15 here), Forest Blue has more white paint (20) than Volcanic Blue (18)

Hence Volcanic Blue will be a darker shade and Forest Blue would be a lighter shade.

(Reason: More white paint means lighter shade & less white paint means darker shade)

Problem: A company is working on two recipes for making coffee

Recipe A uses Cocoa powder & milk in the ratio 3:5

Recipe B uses Cocoa powder & milk in the ratio 5:7.

Find out which method produces a

Strong and concentrated coffee.

Solution: Strong and concentrated coffee means higher concentration of coffee (a greater ratio of coffee to milk).

Now Recipe A: Coffee : Milk = 3:5

Recipe B: Coffee : Milk = 5:7

So we have to compare the ratios of Recipe A and Recipe B.

We can do this by making sure both ratios have same numbers on one side.

Same numbers on left side:-

The ratio of coffee to milk is

Recipe A = 3:5 = 15:25

Recipe B = 5:7 = 15:21

Now 15 parts of coffee has more parts

of milk in recipe A (hence less stronger)

and less parts of milk in recipe B (21)

hence stronger coffee.

So Recipe B will make a stronger coffee

Simplifying a Ratio

Simplest form of a ratio means expressing the ratio in such a form that both the term do not have any common factor other than one (1).

To do this, divide the terms by the HCF (Highest Common Factor).

For example: Simplify 15:25

The HCF of 15 & 25 is 5.

So dividing both terms, we get

15:5:25:53:5.So 3:5 is the simplest form of 15:25.

Very important: A ratio can have decimal or fractions, but we generally simplify it to express in whole numbers.

For example:

1. Simplify $\frac{1}{2}$: 5.

   To make whole numbers, we multiply both number by 2 so we have 1:10.

Theory: We can use ratios to compare two quantities.
For example if 5 marbles cost 20 rupees then the Ratio of marble to price would be
Marble : Price = 5:20 = 1:4
Problem:- Andy and James are preparing for the sports competition. Andy can run 1000 meters in 9 seconds and, James can run 70 meters in 6 seconds.
Use ratios to find which is better prepared?
Solution:- We can use ration to find the better performance
Now ratio of time to distance for Andy would be
Time : Distance = 9:1000

Similarly Time to distance ratio of James would be
Time: Distance = 6:70 = 3:35
Now to compare these ratios, we should make one of the digit common here
So. multiplying James ratio by 3, we get
3x3 : 35x3 = 9:105
Now comparing, we got
Andy
9:100
James 9:105
So James ran 105 meters in 9 seconds as compared to only 100m run by Andy.
Hence James is better.
Problay

Problem: A 5 kg box contain 5 kg of fruits costs $80 and another box containing 8 kg of fruits costs $120. Using Ratios, calculate which is the best deal?

Solution: To find the best deal, we will calculate the ratio of Quantity: Price.
1st box: Quantity: Price = 5:80 = 1:16
2nd box: Quantity: Price = 8:120 = 1:15
It means 1 kg of fruit is available for $16 in 1st box and $15 in 2nd box.
Hence the 2nd box is a better deal.
[we are paying $1 less for each Kilogram of fruits

Concept: How to divide an amount in a ratio

Sometimes we need to divide a total sum into parts as per a ratio.
In such cases, we first add the parts of the ratio. Then divide the total amount by this sum to find the value of one part.
Now multiply the number in the ratio to value of 1 part to find the split.
Common Mistake : You shouldn't divide the total amount directly by the for numbers in the ratio.
to You should add the part first, find the value of one part and use this knowledge to find the value of ratio.

Problem: Chris and Dunken Fores wants to share $9000 in the ratio 2:3. Chris says he will get $4500 (as 9000÷2 = 4500) and Dunken should get $3000 (as 9000÷3 = 3000).
Do you agree with Chris? Explain?

Solution:
Total sum = $9000,
Chris share = $4500

Dunken share = $3000
Sum of Chris & Dunken share = 4500+3000 = $7500
This is not equal to the total sum. Hence Chris is wrong here.
They want to divide in the ratio 3 : 2.
So CHris share = 3 parts
Dunken share = 2 parts
Total sum of ratio = 3 + 2 = 5 parts.
So 5 parts is equal to $9000
So 1 part = 9000 ÷ 5 = $1800.
So Chris share = 3 parts = 3 x 1800 = $5400
Dunken share = 2 parts = 2 x 1800 = $3600
Also we can check the total [5400 + 3600 = 9000]

Problem: A paint mixture is made by mixing Blue and White paint in the ratio 1 : 3. Calculate how much of each colour is needed to make 500 ml of paint?

Solution: Ratio of Blue and White = 1 : 3.
Total paint = 500 ml.
Now sum of parts of ratio = 1 + 3 = 4 parts.
4 parts equal 500 ml of paint.
1 part equals 500 ÷ 4 = 125 ml
So blue paint = 1 part = 125 ml
White paint = 3 parts = 3 x 125 ml = 375 ml
Add to check 125 + 375 = 500 ml of paint.

Type 2: Proportions

Ratio compares two quantities but Proportions compare two ratio.

If two ratios are equal, they are said to be in proportion.

For example: Ben purchase two pastries for $14 and Jack purchased fourpastries for $28.

So Ratio of number of pastries purchased by Ben and Jack = 2 : 4 = 1 : 2

Now the ratio of pastries = Ratio of price

So both ratios are equal & we say they are in proportions .And it means that they both purchased pastries at the same rate. We write proportionas using :: or = sign so here we have 2 : 4 :: 14: 28

Or 2:4 = 14: 28

We read it as "2 is to 4 as 14 is to 28"

why do we need proportions: - Proportions are helpful in situations when we want things to be fair or balanced.

For example: Two friends Alex and Ben purchased 20 marbles for 30 pence. Alex contributed 12 pence and Ben contributed 18 pence.

Now Alex says "divide marbles equally (10 each)" but Ben says "I should get more marbles because I contributed more. you should get 8 marbles and I get 12 marbles".

Now who is correct here?

In such cases, Proportion becomes useful.

Ratio of money given by Alex & Ben = 12:18 = 2:3

Now Alex says divide the marbles (10 each)

So Ratio of marbles with Alex & Ben = 10:10 = 1:1

But Ben says Alex gets 8 marbles & he gets 12 marbles

So Ratio of marbles according to Ben = 8:12 = 2:3.

We see that Ratio of money & Ratio of marbles suggested by Alex do not equal.

But the ratios are same as per Ben.

So we can say that Ben's distribution is correct.

So here 12:18 and 8:12 are in proportion.

This can be written as

12:18 :: 8:12

Or 12:18 = 8:12

and we read it as

12 is to 18 as 8 is to 12.

Example 2: A train travels 25 km in 2 hours. Can the same train travel 50 km in 4 hours?

Solution:- Ratio of distance = 25:50 = 1:2

Ratio of time = 2:4 = 1:2

Two ratios are equal, they are in proportion. Hence the train can travel 50 km in 4 hours.

We can write the result as

25:50 :: 2:4

Terms in a Proportion: A proportion generally has four terms (here 25, 50, 2 & 4).

The first and fourth terms (25 & 4) are known as extreme terms & the second and third terms (50 and 2) are known as middle terms.

Type 2.1: Cross Multiplication

Cross multiplication is a method used to check whether two ratios are in proportion.

Let us check if $\frac{3}{4}$ and $\frac{18}{24}$ form a proportion.

Multiply the Numerator (3) of first fraction to the Denominator (24) of second fraction.

3 x 24 = 72

Now multiply the Denominator (4) of first fraction with the Numerator (18) of second fraction.

4x18 = 72

If the numbers are equal, the ratios form a proportion.

So we can write 3:4::18:24

Cross multiplication means

3x24 = 72 (Product of extreme terms)

Problem: Check whether the ratios 15:18 and 35:42 are in proportion?

Solution:15:18 = $\frac{15}{18} = \frac{5}{6}$ = 5 : 6

Also 35:42 = $\frac{35}{42}$ = 5 : 6

So Clearly 15:18 and 35:42 are in proportion

So we can write 15:18::35:42

Here 15 and 42 are extreme terms & 18 and 35 are middle terms.

Also 15x42 = 630 = 18x35

Product of extreme = Product of middle terms.

Problem: Check whether the following terms are in proportion?

a) 4,6,8,12

b) 33,44,75,100

c) 4,6,8,16

Solution:(a) 4,6,8,12

Ratio of 4 and 6 = 4:6 = $\frac{4}{6} = \frac{2}{3}$ = 2 : 3

Ratio of 8 and 12 = 8:12 = $\frac{8}{12} = \frac{2}{3}$ = 2 : 3

The two ratios are equal, hence the terms are in proportion.

(b) 33,44,75,100

Here ratio of 33 & 44 = 33:44 = $\frac{33}{44} = \frac{3}{4}$ = 3 : 4

Also ratio of 75 and 100 = 75:100 = $\frac{75}{100} = \frac{3}{4}$.

The two ratios are same. So the terms are in proportion.

c) 4, 6, 8, 16.

Ratio of 4:6 = $\frac{4}{6} = \frac{2}{3}$ = 2:3

Ratio of 8:16 = $\frac{8}{16} = \frac{1}{2}$ = 1/2.

The ratios are not equal. Hence the terms are not in proportion.

Problem: Using the cross multiplication method, calculate the missing term if the terms are in proportion?

a) 2:3 = 4:a

b) 1:5 = b:10

Solution: If the terms are in proportions, then we can use cross multiplication rule to check if the product of extreme terms is same as product of middle terms.

a) 2:3 = 4:a

$\frac{2}{3} = \frac{4}{a}$

2 x a = 3 x 4

2a = 12

a = $\frac{12}{2}$ = 6.

b) 1 : 5 = b : 10

$\frac{1}{5} = \frac{b}{10}$

= 1x10 = bx5

b = $\frac{10}{5}$ = 2

Problem: Which of the following statements are true?

(a) 12:18 = 20:30

We have 12 : 18 = $\frac{12}{18} = \frac{2}{3}$

& 20 : 30 = $\frac{20}{30} = \frac{2}{3}$.

Hence 12 : 18 = 20 : 30 (True)

2nd method: It's a proportion. If the product of extreme terms is equal to the product of middle terms.

Here 12:18 :: 20:30

Product of extreme terms = 12 x 30 = 360

Product of middle terms = 18 x 20 = 360

Hence the two ratios are equal and in proportion.

(b) 10:25 and 14:35

We have 10:25 = $\frac{10}{25} = \frac{2}{5}$ (Division by 5) and $\frac{14}{35} = \frac{2}{5}$ (Division by 7) hence it is true.

2nd method: Product of extreme term = 35x10 = 350

Product of middle term = 14x25 = 350

Hence the two ratios are equal.

(c) 7:10 and 14:21

We have 7:10 = $\frac{7}{10}$ (No HCF other than 1)

Also 14:21 = $\frac{14}{21} = \frac{2}{3}$ (Division by 7)

Clearly $\frac{7}{10} \neq \frac{2}{3}$. Hence it is False.

2nd method:-

Product of extreme terms = 7x21 = 147

Product of middle terms = 10x14 = 140

Clearly they are not equal

Hence the ratios are not equal. It's false

Problem: Find the missing number?

(a) 6:? = 12:18

Let us call the missing number be 'x' then 6:x = 12:18 = $\frac{6}{x} = \frac{12}{18}$

Using Cross multiplication, we get

12x=6x18

= x = 108 ÷ 12 = 9

Hence we have 6:9 = 12:18

b) 3:5=?:20

Let's call the missing number 'x'

then 3:5=x:20

= $\frac{3}{5} = \frac{x}{20}$

Using Cross multiplication, we get

5x=3x20=60

x=60 ÷ 5=12

So the expression becomes 3:5=12:20

c) ?:21=4:7

Let's call the missing number be 'x' then x:21=4:7

= $\frac{x}{21} = \frac{4}{7}$

Using Cross multiplication, we have

7x=21x4=84

x=84 ÷ 7=12

So the expression becomes 12:21=4:7

Theory: Division of amount in a ratio

A ratio is a way to compare two or more quantities. Imagine two friends A & B. A has 10 marbles and B has 20 marbles. So we can say ratio of marbles with A & B is

10:20 = 1:2 (Divide by 10)

It means for every 1 marble of A, there are 2 marbles with B.

And we say Ratios can be used to divide money (any quantity).

Suppose you want to divide 100 in the ratio 2:3.

① Add all the parts of the ratio.

It mean add 2+3 = 5 parts.

② Equate total parts to the total money.

So 5 parts = Rs 100

So 1 part = 100 ÷ 5 = 20

So 1 part is equal to Rs 20.

③ Use multiplication to find value of each part. Ratio had 2 part numbers 2 & 3,

So value of 2 part = 20 x 2 = 40

Value of 3 part = 3 x 20 = 60

Problem: Divide $1000 among A, B & C in the ratio 2:3:5?

Solution:- The ratio 2:3:5 has 3 terms but the rules of dividing a number in ratio will remain the same.

We have A:B:C=2:3:5

So first we add all the parts of the ratio

= 2+3+5=10 parts.

These 10 parts are equal to $1000

So 1 part is equal to 1000 ÷ 10 = $100.

Now A will get 2 parts = 2x100 = 200

B will get 3 parts = 3x100 = 300

C will get 5 parts = 5x100 = 500.

So A, B & C will get $200, $300 & $500 respectively.

Type 3: Unitary Method

Unitary method is a method to solve ratio problems by finding the value of 1 unit and then using it to find the value of many units.

For example. If the cost of 5 pencil is $15, then find the cost of 9 pencils.

Here Unitary method says that we first find the cost of one unit (one pen) and then use it to find the cost of many (9 here) units (pens).

Cost of 5 pens = $15

→) Cost of a pen = $15 ÷ 5 = $3.

So cost of 9 pens = $9 * 3 = $27

So the cost of 9 pens is $27.

Consider the situation.

Serena bought 3 burgers for $6. Calculate how much she need to pay if she wish to purchase 5 burgers.

Here we will first calculate the price of one burger and then use this data to find the price of 5 burgers.

Here it is how it is done...

Cost of 3 burgers = $6

So, cost of 1 burger = $6 ÷ 3 = $2.

Hence, cost of 5 burgers = $5x2 = $10.

So here we have used the Unitary method.

Unitary Method: A method in which we calculate the value of one unit first and then use this information to calculate the value of required units.

Problem: If the cost of 8 notebooks is Rs 200, calculate the cost of 5 notebooks? **Solution:-** We can solve this method using the Unitary method and the Proportions method.

Unitary method:- Cost of 8 notebook = Rs 200

=> Cost of 1 notebook = 200 ÷ 8 = 25

=> Cost of 5 notebooks = 25 x 5 = Rs 125.

Proportions Method:-

The number of notebooks and their cost must be in proportions.

Let the cost of 5 notebooks be 'c', then use the method.

Notebook: Price = Notebook: Price

8 : 200 = 5 : c

=> $\frac{8}{200} = \frac{5}{C}$

Using Cross multiplication,

8C = 200 x 5 = 1000

C = 1000 ÷ 8 = Rs 125

So the cost of 5 notebooks is Rs 125.

Problem: A car travels 350 km in 7 litres of petrol. Calculate how much distance the car will cover in 2.5 litres of petrol?

Solution: Using the ratio & Proportion method

The petrol used and the distance travelled must be in proportions

Let the distance travelled in 2.5 litres of petrol be 'D'

So,

Petrol: DIstance = petrol distance

7:350 = 2.5: D

$= \frac{7}{350} = \frac{2.5}{D}$

&xD = 2.5x350 = 875

D = 875 ÷ 7 = 125 km

So the car will travel 125 km in 2.5 litres of petrol.

Unitary Method:

Distance travelled in 7 litres = 350 km.

Distance travelled in 1 litre = 350 / 7 = 50 km.

Distance travelled in 2.5 litres = 50 x 2.5 = 125 km.

Problem: The cost of 15 apples is $50 pence. Use this to calculate how many pens can be purchased in $130.

Solution: Using the method of Ratio & proportions,

Let the number of apples be A which can be purchased with $130.

Now

Apple: price = APple : price

15:50 = A : 130

$\frac{15}{50} = \frac{A}{130}$= 50xA = 15*130

50A = 1950

A = 39

So 39 apples can be purchased from $130.

Unitary Method: Apples purchased in $50 = 15.

So number of apples purchased in $1 = $\frac{15}{50}$

So number of apples that can be purchased in $130 = 130*$\frac{15}{50}$ = 39.

So 39 apples can be purchased.

Problem:- A car travels 50 km in $\frac{2}{3}$ hours.

(a) Calculate the time required to cover 70 km at the same speed?

(b) Find the distance covered in 100 minutes?

Solution: Car travels 50 km in ($\frac{2}{3}$ $hours$ $=$ $\frac{2}{3*60min}$) = 40 minutes

(a) Let the time taken to cover 70 km be 't'.

Now using Ratio & Proportions, we get

Distance : Time = Distance*Time

50 : 40 = 70 : t

$\frac{50}{40} = \frac{70}{t}$

5ot = 70*40

T = 2800÷50 = 56 minutes

so the car would take 56 minutes to cover 70 minutes.

(b) 50 km distance is covered in 40 minutes.

Let the distance covered be "D" in 100 minutes.

Then using Ratio & Proportions,

we have Distance : Time = Distance : Time

50 : 40 = D : 100

$\frac{50}{40} = \frac{D}{100}$

= 40D = 50*100 = 5000

D = 5000 ÷40 = 125km

Problem: A company produces 500 toys in 40 minutes. Calculate the number of toys it will produce in 3 hours.

Solution: Company produces 500 toys in 40 minutes.

So let the toys produced in 3 hours (3x60 = 180 minutes) be T.

Using Ratio & Proportions, we have

Toys: Time = Toys: Time

500:40 = T:180

$\frac{500}{40} = \frac{T}{180}$

40*T = 500*180

40T = 90,000

T = 90000÷40 = 2250

So the company will produce 2250 toys in 3 hrs (180 minutes).

Problem: A printing press prints 12 pages in 20 seconds.

a) Calculate the number of pages it would print in 50 minutes.

b) Calculate the time required to print 150 pages?

Solution: a. while doing problems of ratio and proportion, make sure all units of time (or other variable like distance) are same.

Printing Press prints 12 pages in 20 seconds.

Let the number of pages be 'P' which could be printed in 50 minutes(50*60 = 3000 sec)

Using ratio & proportions, we get

Pages: Time = Page: Time

12: 20 = P : 3000

$\frac{12}{20} = \frac{P}{3000}$

= 20P = $\frac{12*3000}{20} = 1800$

So the printing press would print 1800 pages in 50 minutes.

b) Let say the time required to print 150 pages be T (seconds)

Then using Ratio and Proportions, we have

Pages: Time = Pages: Time

12: 20 = 150: T

$\frac{12}{20} = \frac{150}{T}$

Using Cross multiplication,

12T = 150 x 20

T = $\frac{150*20}{12} = \frac{3000}{12}$ = 250 seconds

So the printing press would take 250 seconds to print 150 pages.

Problem: Venus is making 10 glasses of lemonade. For this, she needed the following ingredients as per a recipe.

3000 ml of water

250 g of sugar

100 ml of lemon juice

a) Calculate the amount of each quantity if Venus wants to make 13 glasses of lemonade.

Solution: a) We will solve this problem, one at a time for each ingredient.

10 glasses lemonade needs 3000ml of water

Let 13 glasses need 'w' ml of water.

Then using Ratio and proportion, we have

Glasses : Water = Glasses : Water

10 : 3000 = 13 : W

$\Rightarrow \frac{10}{3000} = \frac{13}{W}$

⇒ 10W = 13 x 3000

W = $\frac{13}{10}$*3000 = 1.3*3000 = 3900ml

Sugar: Let the sugar needed be 's' grams.

⇒ Glasses : Sugar = Glasses : Sugar

10 : 250 = 13 : s

$\Rightarrow \frac{10}{250} = \frac{13}{s}$

⇒ 10s = 13 x 250

s = $\frac{13}{10} * 250$ = 1.3*250 = 325gm

Lemon Juice: Let lemon juice required be 'L' ml.

⇒ Glasses : Lemon juice = Glasses : lemon juice

10 : 100 = 13 : L

$\Rightarrow \frac{10}{100} = \frac{13}{l}$

⇒ 10L = 13 x 100

L = $\frac{13}{10} * 100 = 1.3 * 100$ = 130ml

Second method:-

Venus was making 10 glasses of lemonade and now she wish to make 13 glasses

So the ratio is

$\frac{13}{10}$ = 1.3

So it means Venus will require 1.3 times of each ingredient used for 10 glasses.

Water = 3000 x 1.3 = 3900 ml

Sugar = 100 x 1.3 = 130 grams

Lemon juice = 100 x 1.3 = 130 ml

Now using Ratio and proportion, it means if the number of glass is increased by 1.3, the ingredients must also increase by a factor of 1.3.

Problem: The ratio of boys to girls in a class is 2:3. If the number of girls is 18, find the total students in the class.

Solution: Ratio of boys to girls = 2:3

Number of girls = 18.

Let the number of boys = B

then Ratio of boys to girls = B:18

Now these two ratios must be equivalent. They must for a proportion.

Equating the two ratios, we get

2 : 3 = B : 18

$\frac{2}{3} = \frac{B}{18}$

3B = 2*18

B = 36 ÷ 3 = 12

So number of boys = 12

Total students = Boys + Girls = 12 + 18 = 30.

ANOTHER METHOD:- Ratio of boys to girls = 2 : 3.

Let the common ratio factor be 'x'.

So number of boys = 2x

Number of girls = 3x

Now number of girls = 3x = 18

X = 18 ÷ 3 = 6

So the number of boys = 2x = 2*6 = 12.

Finding the missing number using a given Ratio.

A ratio is a way to compare two quantities.

Imagine the ratio of boys to girls in a classroom is 3 : 4.

It means there are 4 girls for every 3 boys.

So it can mean any of the following:-

a. The class has 3 boys & 4 girls [Ratio : 4]
b. Class has 6 boys & 8 girls (Ration 3 : 4)
c. Class has 30 boys & 40 girls (Ratio 3 : 4)

It means the actual number of boys and girls can be any number multiplied to the ratio.

So the actual number of boys and girls can be taken as 3x and 4x where x is any number.

If the number of boys is say equal to , then Number of boys = 3x = 15

x = 15 ÷ 3 = 15

Using this value of 'x' we can find number of girls as

4x = 4*5 = 20

So number of boys = 15 & girls = 20

And ratio is 3 : 4.

We will use this concept in solving problems.

So whenever ratio of two quantities is given, the actual number can be obtained by multiplying any number (say x) to both the terms of the ratio.

Problem: An alloy is a mixture of two or more metals to make a new and better material.

Brass is an alloy of zinc and copper mixed in th ratio 2 : 3. If 18 gms of zinc is used, calculate the amount of copper?

Solution: Ratio of zinc and copper = 2 : 3

Amount of zinc = 18 gms

Let the mount of copper be "C"

Then ratio of amount zinc & copper = 18 : C

These ratios (2 : 3) and (18 : C) are equivalent (in proportions), so we have

2 : 3 = 18 : C

$= \frac{2}{3} = \frac{18}{c}$

2c = 18*3 = 54

C = 54 ÷ 2 = 27 gms

So the amount of copper is 42 gms.

Second Method:- Ratio of zinc to copper = 2:3.

The actual amount of zinc & copper can be obtained by multiplying both the number of the ratio by same number.

Let the common multiplier number be x.

So actual amount of zinc = 2x

actual amount of copper = 3x.

Now it is given that amount of zinc

= 2x = 18 gms

x = 18 ÷ 2 = 9 gms

Using this value of x, we get amount of copper = 3x = 3*9 = 27 gms

So amount of zinc = 18 gm, amount of copper = 27 gms

Problem: A fruit punch has orange and pineapple juice in the ratio 5:6. If 30 litres of orange juice was used, calculate how many litres of pineapple juice was used? **Solution:** Ratio of orange and pineapple juice = 5:6

Amount of orange juice = 30 litres

Let the amount of pineapple juice (in litre) be = p.

Then Ratio of amount of orange & pineapple Juice = 30 : p.

Now these two ratios, (5:6) and (30:p) are equivalent (in proportion). So,

5 : 6 = 30 : p

$$\frac{5}{6} = \frac{30}{p}$$

5p = 30x6 = 180

p = 180÷5 = 36 litres

So the amount of pineapple juice is 36 litres

Second Method:- Ratio of orange & pineapple juice = 5:6

The actual value can be obtained by

multiplying both the terms of ratio by same number.

Let the number be x,

then amount of orange juice = 5x

Amount of pineapple juice = 6x

But 5x = 30 (given)

x = 30÷5 = 6

So amount of pineapple juice = 6x = 6x6

= 36 litres.

Problem: The three angles of a triangle are in the ratio 2:3:4. Find the size of each angle.

Solution: The ratio of three angles is 2:3:4.

The actual value of angles can be obtained by multiplying all the terms of the ratio by some number.

Let the common multiplying number be 'x'

then the three angles becomes 2x, 3x and 4x.

Sum of all the angles of a triangle equals 180°

(Angle sum property of a triangle)

=> 2x + 3x + 4x = 180°

9x = 180°

x = 180 ÷ 9 = 20°

So the three angles are

2x = 2 × 20° = 40°

3x = 3 × 20 = 60°

$4x = 4 \times 20 = 80°$

Check ratio of 40:60:80 = 2:3:4

Second method: Ratio of angles = 2:3:4

Sum of all the terms of the ratio = 2 + 3 + 4 = 9

These 9 parts are equal to 180° (Angle Sum property of a triangle)

So

1 part = $180 \div 9 = 20°$

So the three angles are

2 part = $2 \times 20 = 40°$

3 part = $3 \times 20 = 60°$

4 part = $4 \times 20 = 80°$

Problem: The size of three angles of a triangle a, b and c are in the ratio 5:5:8. Calculate the size of all the three angles.

Solution: The three angles of the triangle are in the ratio a:b:c = 5:5:8.

The actual size of the three angles will be obtained by multiplying some number, say (x), to all the terms of the ratio.

Let the common multiplier be x, then the three angles are:

$a = 5x$, $b = 5x$, $c = 8x$.

Now Sum of all angles of a triangle is 180°.

$a + b + c = 180°$

$5x + 5x + 8x = 180°$

$18x = 180°$

$x = 180 \div 18 = 10°$

So the three angles are:

a = 5x = 5 x 10 = 50°

b = 5x = 5 x 10 = 50°

c = 8x = 8 x 10 = 80°

Another method:

Sum of all the terms of ratio = 5+5+8 = 18

These 18 terms are equal to 180°.

So 1 term = 180 ÷ 18 = 10°.

So a = 5x10 = 50°

b = 5x10 = 50°

c = 8x10 = 80°

Problem: The four angles of a quadrilateral are in the ratio 1:2:3:4. Find the size of each angle?

Solution: Ratio of four angles of a quadrilateral = 1:2:3:4

Actual size of the four angles can be obtained by multiplying each term of ratio by same number. Let the common multiplier factor be x.

So the four angles becomes x, 2x, 3x & 4x.

Now x + 2x + 3x + 4x = 360°

[Angle sum property of a quadrilateral]

10x = 360°

x = 360 ÷ 10 = 36°.

So the four angles are

x = 36°

2x = 2×36 = 72°

3x = 3×36 = 108°

4x = 4×36 = 144°

2nd method: The four angles are in the ratio 1:2:3:4

Sum of all the parts of ratio = 1+2+3+4 = 10

These 10 parts are equal to 360°

So value of 1 part = 360°÷10 = 36°.

So the four angles are 36°, 2×36=72°, 3×36=108° & 4×36=144°.

Problem: The angles of a quadrilateral are in the ratio 4:5:7:8. Calculate the size of each of the angles?

Solution: The ratio of angles of a quadrilateral = 4:5:7:8

Let the common multiplying factor be x, then the four angles becomes

4x, 5x, 7x, & 8x.

Now Sum of all the four angles of a quadrilateral is always 360° (Angle sum Property of a quadrilateral).

=> 4x + 5x + 7x + 8x = 360°

=> 24x = 360

=> x = 360÷24 = 15°

So the four angles are

4x = 4x15 = 60°

5x = 5x15 = 75°

7x = 7x15 = 105°

8x = 8x15 = 120°.

So the four angles are 60°, 75°, 105° & 120°.

Concept: How to combine two ratios

Suppose we are given two ratios such that x:y = 2:3 and y:z = 5:6.

Here y is common in both the ratios, but y has different values (3 & 5) in the two ratios.

How to combine these two ratios? To combine the two ratios, make the common variable (y here) has same value in both the ratios.

To do this, we have to find the least common multiple (LCM) of the different values of y (here 3 and 5).

Now LCM of 3 & 5 = 15.

So now we multiply both ratios such that y term becomes 15.

x:y = 2:3 = 10:15.

Also y:z = 5:6 = 15:18.

Now we can combine the ratios as

x:y:z = 10:15:18.

We can also use the above ratio to find

the minimum value of x+y+z

This is x+y+z = 10+15+18 = 43

Problem: The ratio of boys to girls in a playschool to teacher is 2:3 and the ratio of girls to teacher is 7:1. Find the smallest whole number total of boys, girls and teacher combined.

Solution: Ratio of boys: girls = 2:3

and Ratio of girls: Teachers = 7:1.

Now Girls is a common term in both the ratios.

We first combine the two ratios together by making the number of girls same in both ratios

Value of girls in the two ratio is 3 and 7.

Now L.C.M of 3 and 7 is 21. So multiply the above ratios to make number of girls equal to 21, we get

Boys: Girls = 2:3 = 14:21

Girls: Teacher = 7:1 = 21:3

So combining the two, we get

Boys: Girls: Teacher = 14:21:3.

There is no common factor of 14, 21 and 3 other than one. So this ratio is in the simplest form.

So the minimum possible total number of boys, girls and teachers is

14+21+3 = 38

Problem: The ratio of x:y = 3:4 and y:z = 5:6. Find the ratio x:y:z and also find the smallest possible value of x+y+z?

Solution: We are given that,

x:y = 3:4

and y:z = 5:6

Now to make the two ratios same, we have to multiply both ratios to make value of y same. For this we calculate the LCM of values of y in the two ratios.

Now values of y are 4 and 5.

LCM of 4 and 5 = 20.

So multiplying we have

x:y = 3:4 = 15:20

y:z = 5:6 = 20:24.

So now we can combine the ratios as the common y has same value in both the ratios.

So x:y:z = 15:20:24.

Now 15, 20 and 24 have no common factor other than one. Hence

x:y:z = 15:20:24 is the simplest form.

The minimum value of x + y + z = 15 + 20 + 24 = 59.

Problem:- Four number p,q,r, and s are connected as:-

p:q = 2:3

q:r = 6:7

r:s = 21:10

Write their overall ratio p:q:r:s and calculate the smallest possible value of

p+q+r+s?

Solution:- We will solve this problem by taking first two ratios together & then adding third.

We have p:q = 2:3.....

and q:r = 6:7.....

To combine these, we multiply the first

ratio by 2. So it becomes

p:q = 2:3 = 4:6

and q:r = 6:7

Now they can be combined. So,

p:q:r = 4:6:7

Now r:s = 21:10.

To make the 'r' term same, we multiply p : q : r by 3 to get

P : q : r = 4 : 6 : 7 = 12 : 18 : 21

Now combining with r : s = 21 : 10

We get

P : q : r : s = 12 : 18 : 21 : 10.

These terms have no common other tan one, so it is the simplest form of the ratio

The minimum value of p + q + r + s will be 12 + 18 + 21 + 10 = 61

AB: comparing two growing quantities:-

We can use the concept of Ratios to compare two growing quantities.

Problem: Two friends Tim and Paul start money-in with $20 and add $5 every week. Paul starts with $10 and adds $6 every week. Calculate after how many weeks the ratio of their savings would be 13:1 (as equal)?

Solution: Tim starts with $20 and adds $5 per week

So after 'x' weeks, his total saving would be

Total Savings of Tim = Fixed amount + (Amount per week X Number of weeks)

Tim's savings = 20 + 5x

where x is the number of week.

Similarly, Paul's total savings would be

Paul's saving = 10 + 6x

Now we want to find time after which their savings are in the ratio 1:1 (it means the savings are equal).

So equating the two savings, we get 10 + 6x = 20 + 5x

= 10 + 6x - 5x = 20

10 + x = 20

X = 20 - 10 = 10 weeks

So the savings of Tim and Paul would be equal after 10 weeks.

Proof: Tim's saving after 10 weeks

= 20 + 5*10

= 20+50 = $70

Paul's savings = 10 + 6*10

= 10 + 60

= σ$70.\times 10

We will be discussing the concept - "Writing Expressions and formula" in this problem. Imagine two friends A and B are saving money for a party. Let friend A starts with $5 and saves $4 each week, then we can write his total savings after ‘n’ weeks as So after 1 week (n=1), savings = 5 + 4 = $9

After 2 weeks (n = 2), savings = 5 + 8 = $13 & so on.

Similarly say B starts with $15 and saves $3 per week.

So after ’n’ weeks, his saving would be

B = 15 + 3n

After 1 week (n = 1), B = 15 + 3 = $18

After 2 weeks (n = 2), B = 15 + 6 = $21& so on.

If you want to find out when their savings will be equal to each other, and after how many weeks, then we may equate the expressions as 5 + 4n = 15 + 3n

= 5 + 4n - 3n = 15

5 + n = 15

N = 15 - 5 = 10 weeks.

So it means after 10 weeks, their savings would be equal to each other.

Savings after 10 weeks

Savings of A = 5+4010

= 5+40=$45

Savings of B = 15+3010

= 15+30=$45.

So after 10 weeks, the savings of A & B would be equal in the ratio 1:1.

Problem: Emma and Noami are in the habit of collecting postal stamps. Emma starts with 50 stamps, and collects 4 stamps every week. Noami starts with 30 stamps and collects 6 new stamps every week. After how many weeks will their collection be in ratio 1:1?

Solution: We can write expression for total stam collected as:

Total Stamps = Initial Stamps + (Stamps in au x Number of w

So after 'n' weeks, their collection would be

Emma total = 50+4n

Noami total = 30+6n.

To find out after how many weeks their collection would be equal (ratio 1 : 1),

we equate the two expressions

30 + 6n = 50 + 4n

30 + 6n - 4n = 50

30 + 2n = 50

2n = 50-30 = 20

N = 20 ÷ 2 = 10 weeks.

So, Emnia & Naomi will have an equal number of stamps after 10 weeks.

Imagine we are asked to convert the form of 3:15.

Then we write 5:25 in such a way that one term is equal to 3.

So 5:25 = 1:5

Now to convert it to 3:n, we multiply 3 on both sides to get

3:15

Thus we can say that 5:25 = 3:15.

So our n = 15 here

This is known as Unitary method in ratios.

Problem:- The price of 5 apples is 70 pence. Calculate the price of 7 apples.

Solution:- We can use Unitary method here.

Ratio of Apple to price is 5:70, writing in the simplest form, we get

Apple : Price = 1:14

Now multiply the ratio by 7, to get

1:14 = 7:98

Hence 7 apples would cost 98 pence.

Another method:- We can use proportions to solve this problem.

let the price of 7 apples be 'p'.

Now ratio of Apple & Price in form of proportion as

5 : 70 = 7 : p

$= \frac{5}{70} = \frac{7}{p}$

= 5p = 7 x 70

= p = 490 ÷ 5 = 98 pence

So the price of 7 apples is 98 pence.

Problem: Robin went to a money exchange shop and learnt that \(1\) usd is equivalent to 85 Indian Rupee.

$1 = Rs = 85

a) If Robin want to get $65, how many rupees does he need to give?

b) If Robin has Rs 9520, find out how many dollar will he get?

Solution: writing a ratio for Dollar : Rupees, we have the exchange rate as

Dollar : Rupee = 1:85

a) If dollar is $65, then let the rupees be R. So the ratio of dollar & rupee will be 65:R

Now writing the ratio as proportion, we have 1 : 85 = 65 : R

$= \frac{1}{85} = \frac{65}{R}$

= R = 65 x 85 = Rs 5525

(b) Let the number of dollars be 'D'. If the amount of rupees is Rs 9520

So the ratio of dollar to money would be

D: 9520

Now comparing it to the exchange rate of writing in the form of proportion:-

1:85 = D: 9520

$=> \frac{1}{85} = \frac{D}{9520}$

=> 85D = 9520

=> D = 9520 ÷ 85 = 112

So Robin would get $112 in exchange for Rs 9520.

Problem: Express the ratio in the simplified form.

10x: 25x where x is a positive whole number.

Solution: The ratio is 10x: 25x

Since 'x' is a positive number on both sides, so we can divide by 'x' to get

10: 25

Now HCF of 10 and 25 is 5, so dividing both terms by 5, we get

10 ÷ 5 = 2 & 25 ÷ 5 = 5

So 10: 25 = 2: 5.

So the simplified form of 10x: 25x is 2:5

Problem: David and Iga purchased a car for $10,000. David contributed $4000 and Iga contributed $6000.

After 3 years, they sold their car for $6000. They decided to share the money in the ratio in which they contributed to purchase the car. Calculate the money David & Iga will receive?

Solution: Total price of the car = $10,000

David contribution = $4000

Iga contribution = $6000.

Ratio of money contributed by David & Iga would be 4000 : 6000 = 2 : 3.

So for every 2 contributed by David, Iga contributed $3.

Now the car is sold for $6000. So this money must be shared in the ratio 2:3 between David & Iga.

Sum of total number of parts of the ratio 2:3 = 2 + 3 = 5

So these 5 parts are equal to $6000.

So value of one part = 6000 ÷ 5=1200

So amount David will get = 2 x 1200 = $2400 & Iga will get = 3 x 1200 = $3600.

[Check that $2400 + 3600 = $6000]

Problem:- Roger and Rafael team up to play in a Doubles Tennis competition. Roger contributes $900 and Rafael contributes $600 to enter. They win $2500 as prize money. If they decide to share the prize in the same ratio as their contributions, how much money should each player recieve?

Solution: Ratio of money contributed by Roger & Rafael

= 900:600

= 3:2

Sum of parts of the ratio = 3+2 = 5 parts

Now total prize money won = $2500.

So 5 parts is equivalent to $2500

So 1 part is equivalent to 2500 ÷ 5 = $500

So Roger will get 3 parts = 3 x 500 = $1500

& Rafael will get 2 parts = 2 x 500 = $1000

Problem:- Two Cricket players, Steve and Ricky, received a prize money of $6300. They can either share the money in the ratio of their ages or in the ratio of wickets they have taken.

Steve is 12 years old & claimed 9 wickets.

Ricky is 15 years old & claimed 12 wickets.

Find out which ratio, age or wickets, will give Ricky a better share. Explain & check ?

Solution: Steve and Ricky won total prize money = $6300

Now they can either divide this in ratio of their ages or in ratio of number of wickets claimed.

We first check the ratio of ages:-

Steve's age = 12 years

Ricky's age = 15 years

Ratio of their age = 12:15 = 4:5

So Total parts = 4+5 = 9 parts.

Now value of 9 parts = $6300.

=> Value of 1 part = $6300/9 = $700.

So Steve will get 4 parts = 4 x 700 = $2800

Ricky will get 5 parts = 5 x 700 = $3500

Now we check the ratio of wickets taken

Wicket taken by Steve = 9

Wicket taken by Ricky = 12

Ratio of wickets taken by Steve & Ricky = 9 : 12 = 3 : 4.

Sum of parts of ratio = 3+4=7 parts.

So value of 7 parts equals $6300

So value of 1 part = 6300 ÷ 7=\$900

So Steve will get 3 parts = 3 x 900 = $2700

Ricky will get 4 parts = 4 x 900 = $3600.

So we can see that Ricky gets $3500 when prize money is divided in the ratio of age. But he will get $3600 if the prize money is divided in the ratio of wickets claimed. So Ricky will get a better share if prize is divided in the ratio of wickets claimed.

Problem:- Kelvin & Eliud took part in a charity marathon. Their sponsor gave them $1200 as training bonus. They decided to share the amount either in the ratio of the distance they ran during training or the ratio of time spent practicing.

Kelvin trained for 40 km and spent7 hours practicing. Eliud trained for 60km and spent 21 hours practicing.

Find out which ratio, distance covered or time spent will give a better share to Kelvin? Explain?

Solution:- Kelvin and Elind are getting a training bonus of $1200.

Now they can divide this money either in the ratio of distance they ran for training or time spent in practicing.

We will first calculate their share using ratio of distance.

Distance covered by Kelvin = 40 km

Distance covered by Eliud = 60 km

Ratio of distance covered by Kelvin & Eliud = 40:60 = 2:3

Sum of parts of the ratio = 2+3 = 5

So 5 parts equals training bonus of $1200.

So 1 part is equal to 1200 ÷ 5 = $240

So Kelvin will get 2 parts = 2\times 240 = $480

Eliud will get 3 parts = 3 x 240 = $720.

Now we divide the bonus in the ratio of time spent practicing

Kelvin spent 7 hours and Eliud spent 21 hours for practice

So Ratio of time spent = 7:21=1:3

Sum of parts of the ratio = 1+3 = 4 parts

So 4 parts are equal to bonus of $1200

So 1 part equals 1200 ÷ 4 = $300

So Kelvin will get 1 part = $300

Eliud will get 3 parts = 3x300 = $900

So we see that Kelvin gets a better share($480) when bonus is shared in theratio of distance covered in practice.

Problem: A father gives $600 to his daughters to share in the ratio of their ages.

Currently his daughters are aged 5 years old and 10 years old.

Find out how much they will each receive?

In 5 years time, calculate how much the two daughters will receive?

Solution: The father wants to give $600 to his two daughters in the ratio of their ages. The younger daughters are aged 5 & 10 years,respectively.

So ratio of their ape = 5 : 10 = 1 : 2

Sum total parts of the ratio = 1+2 = 3 parts

These 3 parts are equal to $600.

So 1 part must be equal to 600 ÷ 3 = $200

So the younger daughter will get 1 part = $200

and Elder daughter will get 2 parts = 2x200 = $400.

After 5 years, the age of daughters would be

Younger daughter age = 5+5 = 10 yrs.

Elder daughter's age = 10+5= 15 yrs.

So ratio of their age = 10 : 15 = 2 : 3

Sum of total parts of the ratio = 2+3=5

So 5 parts are equal to $600 now.

So 1 part is equal to 600 ÷ 5 = $ 120.

So the Younger daughter will get 2parts = 2x120 = 240.

and Elder daughter will get 3 parts = 3x120 = $360.

So the two daughter will receive after 5 years

Younger = $240

Elder = $360.

Problem:- Express the following ratio in the simplest form?

Solution: Always remember to express the quantities in same units. Also ratio in simplest form should have whole numbers only, no decimal or fractional values.

(a) 13 cm to 2.6 m

Making same units, we get

13 cm to 260 cm [Using1m=100cm]

So 13 : 260

The HCF of 13 and 260 is 13.

So dividing both terms by 13, we get

1 : 20

Hence 13 cm : 2.6 m = 1 : 20

(b) 300 m to 2.7 km

We know that 1 km = 1000 m

So expressing in same units, we get

300 m to 2700 m = 300 : 2700

HCF of 300 & 2700 is 300. So dividing both sides by 300, we get

1 : 9.

Hence 300 m : 2.7 km = 1 : 9.

(c) 25 sec to 3.5 second.

Here units are same. So units just need to express the terms in whole numbers and in simplest form.

Now 25 to 3.5 = 25 : 3.5.

Multiplying both sides by 2, we get

50 : 7.

There is no HCF of 50 & 7 other than 1. So 25 sec to 3.5 = 50 : 7 in simplest form.

Type 4: Miscellaneous Problems

Problem:- The ratio of students to teachers visiting a museum is 6:1. There are 350 visitors in total. If each student ticket cost $1 and each teacher ticket costs twice as much. Find out the total money made from ticket sales ?

Solution:- Total visitors in the museum = 350

Ratio of students to teacher = 6:1

Total parts in the ratio = 6+1=7

So 7 parts are equal to 350

So 1 part is equal to 350/7 = 50

Hence number of students = 6 parts = 6x50 = 300

Number of teachers = 1 parts = 50

Ticket price of each student = $1.

Number of students = 300

So money collected from students = 300x1 = $300

Ticket price of each teachers = $2 (twice as much of student)

Number of teachers = 50

So money collected from teachers = 50x2 = $100

So total money collected from ticket sale = 300 + 100 = $400.

Problem: In a movie theatre, the ratio of children to adults is 2:6. A total of560 tickets are sold.

If an adult ticket costs $10 and a child ticket costs half as much. Find the total money collected?

Solution: Total number of tickets sold = 560

Ratio of Children to Adult = 2:6

Now sum of parts of the ratio = 2+6=8 parts

So these 8 parts are equivalent to 560

So 1 part is equal to 560 ÷ 8 = 70.

Now number of children = 2 parts = 2x70 = 140

Number of Adults = 6 parts = 6x70 = 420

Ticket price of Adults = $10

So money collected from tickets sold to Adults = 10x420 = $4200.

Ticket price of Children = $5 (Half as much as Adults)

Number of children = 140

So money collected from tickets sold to Children = 140x5 = $700.

Total money collected from ticket sales

= 4200 + 700 = $4900.

Problem: In a science club, the ratio of girls to boys is 4 : 5. $\frac{4}{5}$ of the boys are interested in Astronomy as specialization. If there are 135 students in the club, find the number of boys who are interested in Astronomy?

Solution: Total students in Science club = 135.

Ratio of girls to boys = 4 : 5.

Now we calculate the number of boys $ girls.

Sum of all the parts of the ratio = 4 + 5 = 9 parts.

So 9 parts are equal to 135 students.

So 1 parts is equal to 135 ÷ 9 = 15 students

So number of girls = 4 parts = 4x15 = 60

Number of boys = 5 parts = 5x15 = 75

Now $\frac{4}{5}$ of the total boys are interested in Astronomy.

So boys interested in Astronomy = $\frac{4}{5}$x75

= 4x15 = 60.

So 60 boys out of 75 are interested in Astronomy.

Problem: In class IV, the ratio of boys to girls is 4 : 5. $\frac{2}{3}$ of the boys are interested in Football and rest are interested in Cricket. If there are 27 students in the class, find the number of boys interested in Football and in Cricket?

Solution: Total students in class IV = 27.

Ratio of boys to girls = 4 : 5.

Now we calculate the number of boys to girls

Sum of all the parts of the ratio = 4 + 5 = 9 parts.

These 9 parts are equal to 27 ÷ 9 = 3 students

So number of boy students = 4 parts = 4x3 = 12

Number of girl students = 5 parts = 5x3 = 15

Now total boys in class VI are 12.

$\frac{2}{3}$ of boys (12) are interested in Football.

So number of boys interested in Football = $\frac{2}{3}$x12 = 8

Number of boy students interested in cricket = Total - Cricket = 12-8 = 4.

So 8 boys are interested in Football & 4 are interested in Cricket.

Here we discuss a few problems of Ratio & Proportions related to other topics in the Maths syllabus like Geometry, Measurement, Data Handling, Algebra, ARea & volume, Percentage & so on.

Problem: The three angles of a triangle are in the ratio 2 : 3 : 4. Find the size of each angle & what type of triangle it is?

Solution: The three angles of triangle are in the ratio 2 : 3 : 4.

Now sum of all parts of the ratio = 2 + 3 + 4 = 9 parts.

Now these 9 parts must be equal to 180º (Angle sum property of a triangle)

So 1 part must be equal to 180 ÷ 9 = 20º.

Hence the three angles are :-

1st angle = 2 parts = 2x20 = 40º

2nd angle = 3 parts = 3 x 20 = 60º

3rd angle = 4 parts = 4 x 20 = 80º

We see that all the angles are less than 90º, so it is an Acute angled triangle.

And since all the angles are different,

it is a scalene triangle.

Problem:- The three angles of a triangle are in the ratio 1:2:3. Find the size of the three angles & what type of triangle is this?

Solution:- Angles of a triangle are in the ratio 3:4:5

Now sum of parts of the ratio = 3+4+5 = 12 parts

These 6 parts are equal to 180° (Angle sum property of a triangle)

So 1 part must be equal to 180° ÷ 6 = 30°

So the three angles are:

1st angle = 1 part = 30°

2nd angle = 2 parts = $2\times30 = 60°$

3rd angle = 3 parts = $3\times30 = 90°$

Here one of the angle is 90°, so it's a right angled triangle.

Type:- If the ratio of angles is 1:1:1. Find all the angles & type of triangle?

Ratio of angles = 1:1:3

Sum of parts of the ratio = 1+1+3=5parts

So 5 parts are equal to 180^o

1 part = $180^o \div 5 = 36^o$

So the three angles are:

1st angle = 1 part = 36^o

2nd angle = 1 part = 36^o

3rd angle = 3 parts = 36x3 = 108\times 3=108

Here two angles are equal in size, so this is an Isosceles triangle.

Type: Angles in the ratio 1:1:1. Find the angles & type of triangle?

Ratio of angles of the triangle = 1:1:1

Sum of parts of the ratio = 1+1+1=3 parts

These 3 parts are equal to 180^o

So 1 part is equal to $180^o \div 3 = 60^o$.

So the three angles are:

1st angle = 1 part = 60^o

2nd angle = 1 part = 60^o

3rd angle = 1 part = 60^o

All the angles are equal. So this must be an equilateral triangle.

Problem:- The ratio of length to breadth = 7:1. If the perimeter is 64 cm, find the length and breadth & the area of the rectangle.

Solution:- The ratio of length & breadth = 7:1

It means the length and breadth can be some obtained by multiplying a common number to the ratio 7:1

Let the common multiple be 'x'

So length = 7x & Breadth = 1x = x

Now Perimeter = 2 (Length + Breadth)

= 2 (7x + x) = 2×8x = 16x

But, we are given,

16x = 64

x = 64÷16 = 4 cm.

So length becomes 7x = 7×4 = 28 cm.

Breadth becomes x = 4 cm.

Area = Length × Breadth = 28×4

= 112 cm square.