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Type 1: Basics

Expressions and formulas are used to convert everyday situations into mathematical statements.
Expressions contain numbers, variables and Operators
(+, -, *, ÷). Expressions do not contain an "equal" (=) sign.
Equations contain numbers, variables, Operators and an "equal" (=) sign.
For example: $2x + 6$ is an expression.
But $2x + 6 = 12$ is an equation.
Here the letter 'x' is called the variable.
A variable can take any value.
The coefficient of x is 2. Coefficient of a variable is the number multiplied to it.
Here addition (+) is an Operator.

Type 2: Expressions

Problem: Alex is A years old.
Jammik is 5 years older than Alex.
Meera is 3 years younger than Alex.
Gilles is four times Alex's age.
James is Half of Alex's age.
Write an expression for each person’s age?
Solution: We start with information we have. Alex is A years old.
Alex's age = A years
Jannik is 5 years older than Alex, so we add 5 to A.
Jannik's age = (A + 5) years
Meera is 3 years younger than Alex, so we subtract 3 from A.

Meera's age = (A - 3) years
Gilles is four times Alex's age. So we multiple A by 3 \text{
Gilles Age = 3*A years
= 3A years
James is half of Alex's age. So we divide A by 2 (or multiply by $\frac{1}{2}$).
James's age = A ÷ 2 = $\frac{A}{2}$ years.

Problem: Yelena has y marbles... Two has 6 more marbles than Yelena. Paul has 5 fewer marbles than Yelena. Victor has double the number of Yelena's marble. Newton has one-third of Yelena's marble.

Solution: We start with information we have.
Number of marbles with Yelena = y
Ivo has 5 more marbles, so add 6 to y.
Ivo marbles = y+6.
Paul has 5 fewer marbles than Yelena, so subtract 5 from y.
Marbles with Paul are y-5.
Victor has double the Yelena's marbles, so multiply y by 2.
Marbles with Victor are 2*y = 2y
Newton has one-third of Yelena's marbles, so divide y by 3. (or multiply y by $\frac{1}{3}$). Marbles with Newton are y ÷ 3 = $\frac{y}{3}$.

Problem: Steve has a bag containing n apples. Write expressions for the total number of apples if:
a) He buys 3 more apples and then ate 4 apples.
b) There are 4 apples.

Solution: Steve has n apples initially.
a) He buys 3 more apples: So we add 3 to n.
New total = n+3
He ate 4 apples, so we subtract 4 from (n+3).
Final total = (n+3) - 4
= n+3-4
= n-1
For example let initial apples n = 10.
Buys 3 more, total = 10+3 = 13.
Ate 4 apples, New total = 13-4 = 9.
Our final expression also says the same.
Final total = n-1.

If n=10, n-1 = 9.

Problem: Albert has x books. Write an expression for the number of books, if:

a). He buys three times the number of books. And then

b). Donate 3 books to a library.

Solution: Albert has initially x books.

a). He buys 3 times the number of books. Three times of x = 3x

So total = x + 3x = 4x

b). Donate 3 books to library. So subtract 3 from last total.

Final books = 4x - 3.

Example:- Let the number of Books X \ 5

Three times of x = 3*5 = 15

So total = 5 + 15 = 20

Donate to library = 3

Final total = 30 - 3 = 17

Our final expression is 4*-3

Taking X = 5, we get

Total = 4*5 - 3 = 20 - 3 = 17

Problem:- Pauli thinks of a number x. Write expression for the following cases.

a. Multiply the number by 4, then add 2.

b. Divide the number by 3, then subtract 5.

c. Multiply by 2, then subtract 9.

d. Divide by 5 and add 5.

e. Multiply the number by 8, then subtract the result from 50.

Solution:- Initial number = x.

a. 4x + 2
b. $\frac{x}{3} - 5 = x \div 3 - 5$
c. 2x - 9
d. (x ÷ 5) + 5 = $\frac{x}{5}$ + 5
e. Multiply by 8 = 8x.
   Subtract the result form 50.
   Final result = 50 - 8x.

Problem: Galileo thinks of a number x. Write an expression for the numbers obtained if he:

a.) Multiplies the number by 5, then adds 2.

b.) Multiplies the number by 3, then subtracts 4.

c.) Divide the number by 2, then adds 5.

d.) Check that the expressions are correct by taking any value of the variable x.

Solution:

Initial number = x a. Multiply by 5, then add 2 New number = 5x + 2	Let the variable x = 4 4*5 + 2 = 22 5x + 2 = 5*4 + 2 = 22
b. Multiply by 3, then subtract 4 c. New number = 3x-4	4*3 - 4 = 12 - 4 = 8 3*4 - 4 = 12 - 4 = 8
d. Divide by 2, then add 5 New number = x ÷ 2 + 5 $= \frac{x}{2} + 5$	4 ÷ 2 + 5= 2 + 5 = 7 $\frac{4}{2} + 5 = 2 + 5 = 7$

Problem:- Agustin goes to a stationary shop. A pen costs x and a pencil costs y. Write an expression for the total cost of

(a) One pen and one pencil.

(b) 3 pens and 2 pencils.

Solution:- Cost of one pen = x.

Cost of one pencil = y.

(a) Cost of one pen & one pencil is

x + y = (x+y).

(b) Cost of 3 pen =3x

Cost of 2 pencil = 2y

Total cost = 3x + 2y.

Problem: Choosing your own letters, write expressions for these situations:-

a. Total cost of 3 pastries and 5 Burgers.
b. Total cost of 2 Pasta and 3 cutlet.
c. The cost of 7 notebook is doubleled.
d. The total value of 4 diamond is doubled.
e. The length of 12 ribbon is tripled.

Solution: a. Let's say the cost of one pastry is 'p' and one Burger is 'b'.

Cost of 3 pastries = 3*p = 3p

Cost of 5 Burger = 50*b = 5b

Cost total cost C = 3p + 5b.

b. Lets take the cost of one pasta ne 'p' and one cutlet be 'c'.

And one cutlet be 'c'.

Cost of 2 pasta = 2*p = 2p.

Cost of 3 cutlet = 3*c = 3c.

Total cost = 2p + 3c.

c. Let the cost of one notebook be 'n'.

Cost of 7 notebook = 7*n = 7n.

Cost of 7 notebook is doubled = 2*(7n) = 14n.

d. Let say the value of one diamond = 'd'.

Value of 4 diamond = 4*d = 4d

Value is doubled = 2*(4d) = 8d.

e. Let say the length of one ribbon be 'r'.

Length of 12 ribbons = 12*r = 12r.

Length is tripled = 3*(12r) = 36r.

Problem: Write for the following:-

a. 'A' more than 'b'.
b. 'E' less than 'f'.
c. 15 more than x.
d. x more than 3 times y.
e. P less than two times q.
f. 5 times 'm' multiplied by 'g'.

Solution: a. A more than b.

It means 'b' is increased by 'a'.

So expression = a + b.

b. 'E' less than 'f'

Means 'f' is reduced (decreased) by 'e'

Expression = f - e.

c. 15 more than x.

Expression = x + 15.

d. 'X' more than 3 times y.

e. P less than two times q.

Two times q = 2q

P less than 2q = 2q - p

Expression = 2q - p

F. 5 times 'm' multiplied by 'g'

5 times m = 5m.

G multiplied by 5m = g*5m

= 5mg

= 5gm.

Problem: A small rectangle has a width of m cm and its length is five times the width. A large rectangle has a width of n cm and its length is 2 more than twice the width. Write an expression for the:

a) Perimeters of both the rectangles
b) Difference in length of the two rectangles

Solution (a): We consider the two rectangles separately.

Small rectangle

Width = m

Length = five times the width =

5 x m = 5m

Perimeter means the sum of all the sides or the length of the boundary of a shape.

Perimeter of small rectangle: = 2 length + 2 width

= (2*m)+ 2*(5m)

= 2m + 10m

= 12m.

Large Rectangle

Width = n.

Length = 2 more than twice the width

= 2 + 2n.

Perimeter of large rectangle = (2*length) + (2*width)

= (2*n) + 2 *(2+2n)

= 2n + 4 + 4n

= 6n + 4

b. Difference of the length of the two rectangles = length of large rectangle - length of small rectangle = 5m - (2n + 2).

Problem - Which of the following are expressions and which are equations?

(a) 2x + 5

(b) 3y + 6 = 9

(c) 2t = 5

Solution: We know that expressions contain only numbers and variables. Expressions do not contain an equal to (=) sign.

Equations contain numbers, variables and equal to (=) sign.

(a) 2x + 5 is an expression. (No “equal to” sign)

(b) 3y + 6 = 9 is an equation. ("Equal to” sign)

(c) 2t = 5 is an equation.

Problem: A thread of 12 cm length is joined to a thread of x cm and a thread of y cm. Write an expression for the total length of the new thread formed?

Solution: We have three threads.

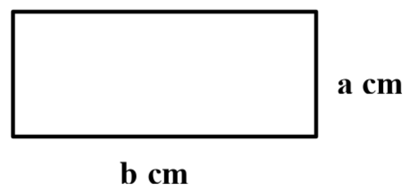
Length of thread 1 = 12cm

Length of thread 2 = x cm
Length of thread 3 = y cm
Total length means all the threads are joined together.
Total length = (12 + x + y) cm
Note: Always mention unit in the answer.

Problem: Sita has 7 ribbons of length x cm, and an another ribbon of length 5 cm. Write an expression for the total length of all the ribbons.

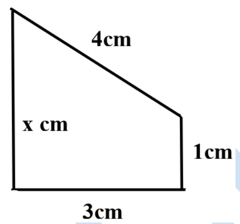
Solution: Sita has 7 ribbons of length x cm each.
So total length = x + x + x + x + x + x + x
Total length = 7x cm.
Another ribbon of length 5 cm.
Total length = (7x + 5) cm.

Problem: Find an expression for the perimeter this rectangle:



(a)

Solution: Perimeter is the sum of all the sides of a figure.
Perimeter = sum of length of all sides
= a + a + b + b
= (2a + 2b)cm
= 2(a + b)cm

b.


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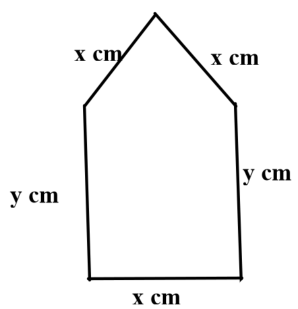
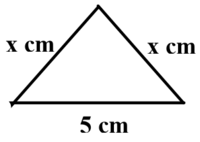


d. 

Solution: b. Perimeter (P) = 1 + 3 + 4 + x = (8 + x)cm
c. Perimeter (P) = (x + x +y + y + y)cm = (2x + 3y)cm
d. Perimeter (P) = (x + x + 5)cm
= (2x + 5)cm
Note:- Always mention units in the final answer.

Type 3: Evaluating expressions

Problem: Find the value of each expression:-

a. X + 5 when x = 2
b. Y - 3 when y = 8
c. X + y when x = 5 and y = 3
d. 5x - 2y when x = 3 and y = 5
e. $\frac{x}{5}$ when x = 30
f. $\frac{22}{x}$ - 1 when x = 11
g. $\frac{a+b}{4}$ - 2 when a = 13, b = 3.

Solution: a. X + 5 when y = 8
Substitute x = 2
So x + 5 = 2 + 5 = 7
b. Y - 3 if y = 8
Put y = 8,
Y - 3 = 8 - 3 = 5.
c. x + y when x = 5 and y = 3
Put x = 5 and y = 3, we get
X + y = 5 + 3 = 8
d. 5x - 2y when x = 3 and y = 5
Put x = 3, y = 5, we get
5x - 2y = 5*3 - 2*5
= 15 - 10 = 5
e. $\frac{x}{5}$ when x = 30

Put x = 30, $\frac{x}{5} = \frac{30}{5}$ = 6

f. $\frac{22}{x} - 1 = \frac{22}{11} - 1$ = 2 - 1 = 1

g. $\frac{a+b}{4} - 2$ when a = 13, b = 3

Put a = 13, b = 3

$\frac{a+b}{4} - 2 = \frac{13+3}{4} - 2 = \frac{16}{4} - 2$ = 4 - 2 = 2

Problem: Write a formula for the following cases. Use letters of your choice.

a. Formula for number of days in any number of weeks. Use the formula to calculate number of days in two weeks.
b. Number of minutes in any number of hours. Use it to find number of minutes in 3 hours.

Solution: a. We know that there are 7 days in a week.

Number of days = 7*Number of weeks.

Let's take d for days and w for weeks, so we have

D = 7*10 = 70

Number of days in 2 weeks.

So w = 2

Hence d = 7*w = 7*2 = 14 days.

b. There are 60 minutes in an hour.

Number of minutes = 60*number of hours.

Let's take m for number of minutes and h for number of hours. Then

M = 60*h = 60h

If number of hours = 3

H = 3

So m = 60h = 60*3 = 180 minutes.

Problem: Rebecca and her four classmates buy a cake together. Write a formula to find out how much each person pays in :-

a. Words b. Letters

c. Use the formula to find the amount paid by each person if the total cost is $120

Solution: Total persons = 5 (Rebecca & four classmates)

So the amount each persons pays is

a. Amount each person pays = Total cost ÷ Number of people
b. Let 'a' is the amount each person pays & 't' is total cost

   Then a = t ÷ 5 = $\frac{t}{5}$
c. If the total cost 't' = $120

Then 'a' = $\frac{120}{5}$ = 24

So each person has to pay $24.

Problem: Eva sells handmade greeting cards for $15 each. She writes the formula as

M = 15 G

a. What do M and G stand for?
b. Write the formula in words?
c. Calculate M if G = 5?

Solution: M = 15 G

a. Here M stands for total Money collected and G stands for greeting cards sold.
b. Total money = 15*Number of Greeting cards sold.
c. If G = 5, then

   M = 15*5 = $75

   So Eva gets $75 id she sells 5 greeting cards

Problem: Aarav uses the formula R = T-S to calculate the money remaining with him.

R = Remaining money

T = Total salary

S = Shopping expenses

a. Find the value of R if

   T = $150 and S - $120

b. Find T if R = $50 and $60.

Solution: We have the formula

R = T-S

If T = $150 and S = $120

a. If T (total income) = $150

   And s (shopping expense) = $120

   Then R(Remaining money) = 150-120 = $30

   So the remaining amount R = $30.

b. We have R = T-S

   If we want T, we should add S to both sides

   R+S = T-S+S

   = R+S = T

   Or T = R+S

Now R = $50 and s = $60

So T = 50+60 = $110.

Problem: A school uses the formula. C = S + J to calculates the total cost of snacks for a farewell party.

A student thought that s is the cost of a sandwich and j is the cost of a juice.

a. Explain why the student could be correct?
b. Explain why the student might be wrong?
c. Suggest a better formula if 3 sandwich and 4 juices are ordered.

Solution: Formula is

C = S + J

Student thinks S is the cost of a sandwich & J is the cost of a juice.

a. Student could be correct if only one sandwich and one juice is ordered.
b. However S and J are not clearly defined.

   S and J can be number of sandwiches & juices ordered, instead of their prices.

   Or maybe there are more than one sandwich or juice is ordered:- then the formula would need multiplication.

   So student might be wrong if S and J don't mean the cost of one sandwich or a juice.

Problem: James calculates the amount of money left using the formula:-

M = x - y

His friend Rafael thinks that x is the total expense and y is the total salary.

a. Is Rafael correct? Justify your answer?
b. How can you improve James formula?

Solution: The formula is

M = x-y

Where M is the amount of money left and 'x' and 'y' are not specified

a. Rafael thinks that

   X = expenses

   Y = salary

   Then formula would be

   M = expenses - salary

   But this doesn't make sense because money left is (salary - expenses).

   Hence Rafael is incorrect.

b. We could improve James formula by using better letters.

   M = S - E

   Where M = money left

   S = salary

   E = expenses.

Problem: If x - y = y - x, what can you say about x and y?

Solution: If x-y = y-x

Adding y to both sides, we get

X = 2y-x

Now adding x to both sides, we get

2x = 2y

Dividing both sides by 2, we get x = y.

So x = y, x is equal to y.

If x is not equal to y (x ≠y), then (x-y) can never be equal to (y-x).

Example:- Let's take x = 5 and y = 2

Then x - y = 5 - 2 = 3

And y - x = 2 - 5 = -3

Clearly 3 is not equal to -3.

Problem: If x ÷ y = y ÷ 3, what can we say about x and y? Can x and y be zero?

Solution: We have x ÷ y = y ÷ x

$= \frac{x}{y} = \frac{y}{x}$

Now we can solve a head by two ways.

$\frac{x}{y} = \frac{y}{x}$ Doing cross multiplication, we get $x^2 = y^2$	Multiply both sides by (x*y), we get $\frac{x}{y}$*(x*y) = $\frac{y}{x}$*(x*y) = $x^2 = y^2$

So the square are equal. It means that either:-

a. x = y or,

b. x = -y

[Remember:- (-y)*(-y) = y^2, negative sign cancels.]

No, x and y cannot be zero because we cannot divide by zero.

Problem: Calculate the value of the expressions:-

a. 2x+4y-3z where x = 4, y = 2 & z = 1.

b. $3ab + \frac{c}{2}$ where a = 3, b = 2 and c = 4.

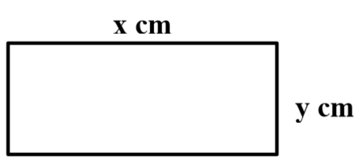
Solution: $3ab + \frac{c}{2}$

$= (3*3*2) + \left(\frac{4}{2}\right)$

= 18 + 2 = 20

Type 4: Simplifying Expressions/ Collecting Like Terms

Simplifying expressions or collecting like term means combining terms that are alike.
Imagine you have 3 red marbles and 2 red marbles.
So total number of red marbles = 3 + 2 = 5 red marbles.
It means combining terms which have same variable.
Example 1
In maths, we write this as:
$3x + 2x = 5x$
Example 2:
Imagine you have 3 apples + 2 apples + 4 bananas.
We cannot add apples to bananas. We can only add apples to apples.
So if a = apples & b = banana.
then 3 apples + 2 apples + 4 banana
$= 3a + 2a + 4b$
$= 5a + 4b$
Example: Imagine you have 5 pencils and 3 erasers. One of your friend asks for 1 pencil and 1 eraser, then how many are left with you.
Let pencil = p and eraser = e.
Then you have $5p + 3e$.
Friend asked for $1p + 1e$.
So left with you will be
$5p + 3e - 1p - 1e$
$5p - 1p + 3e - 1e$
$= 4p + 2e$
[We can add or subtract terms with same letters (variable).]
Problem: Write two expressions for the perimeter of a rectangle.



Solution: Perimeter = Sum of length of all sides.
$= x + x + y + y$
$P = (2x + 2y)$ cm
In addition, the order does not matter.

So P = (2y + 2x)cm

Problem: Simplify by collecting like terms:

a. x + 3x + x

b. 3y + 6y - 9y

c. x + y + y + x

d. a + 4b + 6a - 2b

e. 5x - y + 2y + 5x + y - y

Solution:

a. x + 3x + x = 5x

b. 3y + 6y - 9y = 9y - 9y = 0y (or just 0)

c. x + y + x + y = x + x + y + y = 2x + 2y

d. a + 4b + 6a - 2b = a + 6a + 4b - 2b = 7a + 2b

e. 5x - y -y = 5x + 0y

Rules: Always remember,

$x^2 = x*x$

$y^3 = y*y*y$

Also

$x^2 + 2x^2 = 3x^2$

$2y^3 + 4y^3 = 6y^3$

$x^2 + 2x^2 + 3x = 3x^2 + 3x$

[We cannot $3x^2 + 3x$, they]... are not like terms)

Problem: Simplify:

a. 3x + 4 - x + 2

b. 2x - 3y + 3x - 4y

c. 7a + 3b - 4c - 2b - 3c - 9a

d. $3x^2 - 4x + 2x^2 + 2x + 2$

e. $2x^2 + 2x + 2 - 3x^2 - 4x - 5$

Solution: We will solve using 'collecting like terms'.

So we have:

a. 3x + 4 - x + 2= 3x - x + 4 + 2 = 2x + 6

b. 2x - 3y + 3x - 4y = 2x + 3x - 3y - 4y = 5x - 7y

c. 7a + 3b - 4c - 2b - 3c - 9a = 7a - 9a + 3b - 2b - 4c - 3c = -2a + b - 7c

d. $3x^2 - 4x + 2x^2 + 2x + 2 = 3x^2 + 2x^2 - 4x + 2x + 2 = 5x^2 - 2x + 2$

e. $2x^2 + 2x + 2 - 3x^2 - 4x - 5 = 2x^2 - 3x^2 + 2x - 4x + 2 - 5 = -x^2 - 2x - 3.$

Problem: Express the following as simple fraction:-

a. $\frac{x}{3} + \frac{8x}{3}$

b. $3x - \frac{x}{3}$

c. $\frac{x}{2} + \frac{x}{3}$

d. $\frac{x}{5} + \frac{3x}{4}$

e. $\frac{2x}{5} - \frac{x}{3}$

Solution: a. $\frac{x}{3} + \frac{8x}{3} = \frac{x+8x}{3} = \frac{9x}{3} = 3x$

b. $3x - \frac{x}{3} = \frac{9x}{3} - \frac{x}{3} = \frac{9x-x}{3} = \frac{8x}{3}$

c. $\frac{x}{2} + \frac{x}{3} = \frac{3x+2x}{6} = \frac{5x}{6}$

d. $\frac{x}{5} + \frac{3x}{4} = \frac{4x+15x}{20} = \frac{19x}{20}$

e. $\frac{2x}{5} - \frac{x}{3} = \frac{6x-5x}{15} = \frac{x}{15}$

Type 5: Multiplying a constant over a bracket

We know that 2x = x + x

Similarly, 2(x + 5) means

2(x+5) = (x+5)+(x+5)

= x + 5 + x + 5

= 2x + 10

So, 2(x+5) = 2x + 10

Basically, x and 5 are multiplied by 2 separately.

Problem: Expand the following expressions

(a) 2(x + 1)
(b) 3(y + 6 − x)
(c) 5(2x + y − 5)
(d) 4(2p + 6q − 5r)

Solution: a. 2(x + 1) = 2x + 2

b. 3(y + 6 - x) = 3y + 18 - 3x

c. 5(2x + y - 5) = 3y + 18 - 3x

d. 8p + 24q - 20r

Problem: Roger has x chocolates for the school party. Novak has 5 more chocolates than Roger. Andy has triple the number of chocolates that Novak has.

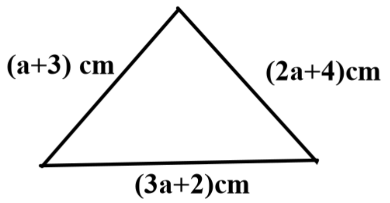
a. In terms of x, write expressions for the number of chocolates that Rose, Novak, and Andy has.
b. Write an expression for the total number of chocolates all three friends.

Solution: a. If Roger has ‘x’ chocolates and Novak has 5 more chocolates than Roger, then Novak has (x + 5) chocolates.
Andy has three times the number of chocolates with Novak.
So Andy has 3 × (x + 5) = 3(x + 5) = 3x + 15 chocolates with him.
b. Total number of chocolates = x + x + 5 + 3x + 15
= 5x + 20
= 5(x + 4)

Problem: Please fill in the blanks using whole numbers to make the algebraic expressions true.
(a) 5x + 15 = (_)(__x + __)
(b) 16x − 36 = (_)(__x − __)
(c) 54 − 18y = ()(− ___y)

Solution: In such problems, we take out the any greatest common factor (GCF) outside the bracket.
(a) 5x + 15. The factors is only 5 of 5 and 15.
So we can write
5x + 15 = 5 (x + 3)
= (5) (1x + 3)
(b) 16x - 36. Factors of 16 are 2,4,8 and 16.
Factors of 36 are 2,4,9,12
Highest common factors is 4
Using 4 as common factor, we can write (16x - 36) = 4(4x - 9)
© 54 - 18y
Highest common factors of 54 are 18 is 18.
So 54 - 18y can be written as
54 - 18y = 18(3 - y)

Problem: Write an expression for the perimeter of the triangle. Write the expression as simply as possible.



Solution: Perimeter is the sum of length of all the sides.
So perimeter = (a+3) + (2a+4) + (3a+2)

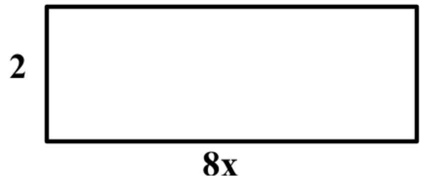
= a + 3 + 2a + 4 + 3a + 2
= (6a + 9)cm.
We can simply it further.
Highest common factor (HCF) of 6 & 9 is 3.
So (6a+9) = 3(2a+3) cm.

Problem: a. The length of a rectangle is 4cm and its breadth is (2a+3)cm. Find an expression for the area and perimeter of this rectangle.

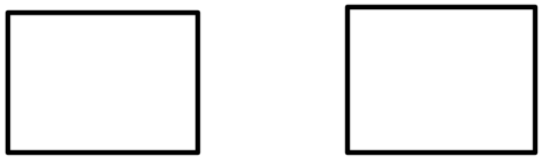
b. Can you think of a rectangle having some area as the above rectangle. Do both the rectangle have the same perimeter?

Solution: length of rectangle = 4cm
Breadth of rectangle = (2a+3)cm.
We know Area = Product of length and Breadth
= 4*(2a+3)
= (8a+12) cm^2.
Perimeter = sum of lengths of all the sides.
= length + length + breadth + breadth
= 2(length + breadth)
= 2[4+(2a+3)]
= 2[4+2a+3]=2(2a+7) = (4a+14) cm
b. Yes, we can think of a rectangle with same area as before.
Let's choose length = 2cm & breadth = (4a+6)cm
So area Area = length*breadth
= 2*(4a+6)
= (8a+12)cm^2
Also Perimeter = 2(length + breadth)
= 2(2+4a+6)
= 2(8+4a)
= (16+8a) cm
We see that we can have different rectangles with same area but different perimeters.

Problem: A rectangle have the following dimensions.



a. Write an expression for the area and perimeter of this rectangle.
b. If the rectangle is divided into two parts as shown.



Then check whether the sum of area and perimeter of these smaller rectangles is equal to the area and perimeter of the larger rectangle.

Solution: Perimeter of large rectangle - sum of length of all the sides.

= (2+8x+2+8x) cm

= (4+16x)cm

= 4(1+4x)cm

Area of a rectangle = Product of its length & breadth

= 2*8x

= 16x cm^2

b. Rectangle is divided into equal half such that

Length = 8x ÷ 2 = 4x & breadth = 2cm

So perimeter of one small rectangle = sum of length of its sides.

= 2+4x+2+4x

= 4+8x

= 4(1+2x).

Perimeter of both the rectangle = 2*(4+8x) = (8+16x)cm.

Area of one small rectangle = length*breadth = 2*4x = 8x

So sum of area of both rectangle = 2*8x = 16x cm^2

We see that clearly the sum of of both the smaller rectangle is equal to the larger rectangle.

But the sum of perimeter of smaller rectangle is more than the original rectangle.

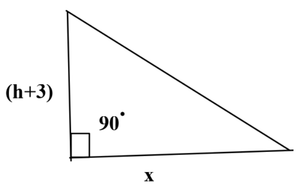
Multiplying a constant over a bracket. The area of a triangle when its base and height are known is

Area of a triangle = $\frac{1}{2}$*Base*Height.

Problem: A right angled triangle has an area of 3h+a square units. If the base is x units length & the height is (h + 3) units, find the value of x?

Solution: We know that the Area of a right angled triangle is

Area = $\frac{1}{2}$*Base*Height



So we have (3h + a) = $\frac{1}{2}$*x*(h + 3)

Multiplying 2 on both sides,

We get

6h + 18 = x(h +3)

Now we can write 6h + 18 = 6(h + 3)

So we have x(h + 3) = 6(h + 3)

Dividing both sides by (h + 3), we get

X = 6 units.

Problem: The are of a right angled triangle is (7k + 28) square units. If the base is y units and height is (k + 4) units, find the value of y?

Solution: Area of a right triangle = $\frac{1}{2}$*Base*Height

$\frac{1}{2}$*y*(k + 4) = 7k + 28

Multiplying both sides by 2,

We get

y *(k + 4) = 14k + 56

Now we can write 14k + 56 = 14(k + 4)

So, we have y(K+4) = 14(K+4).

Dividing both sides by (K+4), we get

y = 14 units.

So the value of base of the triangle is 14 units.

Problem: The perimeter of a rectangle is 12x + 24 units. If the length of the rectangle is 8 units, find an expression for its breadth and area?

Solution: Perimeter = 12x + 24

We know that

Perimeter = 2(Length + Breadth)

Perimeter = 2(8 + b)

where b is the breadth of the rectangle.

Now

2(8 + b) = 12x + 24

= 16 + 2b = 12x + 24

Subtracting 16 from both sides, we get

2b = 12x + 8

Dividing both sides by 2, we get

b = 6x + 4

So the breadth is (6x + 4) units.

Now Area of a rectangle = Length*Breadth

= 8 * (6x + 4)

= 48x + 32 square units

Problem: The perimeter of a rectangle is (10y+6) units and one of the side (length) is 7 cm. Find an expression for the area of the rectangle?

Solution: We know the Perimeter is the sum of the length of the boundary.

For a rectangle,

Perimeter = 2 (Length + Breadth)

We are given that

10y + 6 = 2(7 + b)

Where b is the breadth of the rectangle.

10y + 6 = 14 + 2b.

Subtracting 14 from both sides, we get

2b = 10y + 6 - 14 = 10y - 8

Dividing both sides by 2, we get

Now Area of a rectangle = Length*Breadth

= 7*(5y - 4)

= (25y - 28)sq units.

Problem: Write a formula in words to find the number of chairs needed if each desk requires four chairs and we know the number of desks. Also write the formula using letters and symbols.

Solution: The formula in words would be:

Number of chairs needed = Number of desk*4.

Using 'c' for number of chairs and 'd' for number of desks, we get,

C = d*4

Or c = 4d

Problem: Write a formula in words and symbols that will allow you to find the number of books on a shelf if each shelf has 15 books and you know the number of shelves.

Solution: The formula in words is:

Number of books = Number of shelf*15

If we use 'b' for number of books and 's' for number of shelves, then

B = s*15

Or b = 15s

Problem: Write a formula in words and symbols that express the bottom number in terms of the top number.

a.

2	5	7	9

5	7	9	12

b.

2	5	7	9
8	20	28	36

c.

6	10	14	16
3	5	7	8

d.

4	6	7	9
16	36	49	81

e.

2	3	4	5
5	10	17	26

Solution: (a) The bottom number is equal to the top number plus 3.

bottom number = top number + 3

$b = t + 3$

(b) Bottom number is equal to top number multiplied by 4.

Bottom number (b) = Top number (t)*4.

$= b = t*4.$

$b = 4t.$

(c) Bottom number is equal to top number divided by two.

Bottom number (b) = top number (t) ÷ 2.

$= b = t \div 2$

Or $b = \frac{t}{2}$

(d) Bottom number is equal to top number multiplied with itself.

or

Bottom number is equal to the square of the top number.

Bottom number (b) = Top number (t)*Top number (t)

$= b = t*t$

$= b = t^2$

(e) Bottom number is equal to the top number multiplied with itself and then add one.
Bottom number is equal to the square of the top number plus one
Bottom number (b) = Top number (t) * Top number (t) + 1

Problem: The time in London is 5 hours behind the time in Tokyo. Write a formula in words and symbols liking the time? Also calculate what time it is in London if it is 1 pm in Tokyo?

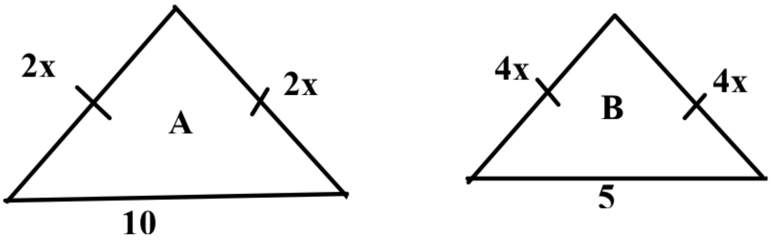
Solution: London time is 5 hours behind (-) the Tokyo time.
So time in London = Tokyo Time - 5 hours
Or L = T - 5
If Tokyo time = 1pm
Then time in London will be
London time (L) = T(1 pm) - 5 hours
= 8 am.
So the time in London is 8am when it is 1pm in Tokyo.
Reason:- 1. Subtract at 1pm.
2. Subtract 1 hour → 12 noon.
3. Subtract remaining 4 hour → 8am

Problem: The time in New Delhi is 4 hours ahead of the time in Washington. Write a formula in words and symbol linking the two times. If the time in Washington is 11 pm, calculates the time in New - Delhi?

Solution: Time in New Delhi is 4 hours ahead (+) of the time in Washington
So time in New Delhi (N) = Washington time/w) + 4
Or N = W + 4 hrs
If time in Washington is 11 pm, then New Delhi time would be
N = W + 4
= 11am + 4hrs
= 3pm
So time in New Delhi is 3 pm when it is 11 am in Washington.
Reason: a). Start at 11 am
b). Add 1 hour → 12 noon
c). Add remaining 3 hrs → 3 pm.

Type 6: Advanced Problems

Iga and coco are studying two triangles A and B.



a. They calculate that perimeters of triangle A is P(A) = 4x+10 and perimeter of triangle B is P(B) = 8x+5. Check whether they are correct?

b. Iga says that the perimeter of triangle A will always be greater than the perimeter of triangle B because "10" in making a triangle is no bigger than '+5'.
However, Coco says that the perimeter of triangle B is still always be larger than perimeter of A because it has a '8x' in the formula which is bigger than '4x'?
Explain which one is correct.
Ans Perimeter = sum of the lengths of all the sides
Triangle A has sides 2x, 2x and 10.
So, Perimeter (A), P(A) = 2x + 2x + 2 + 10
= 4x + 12
Triangle B has sides 4x, 4x and 5.
So, P(B) = 4x + 4x + 5
= 8x + 5
b. Iga says that the perimeter of A will always be greater than the perimeter of B because '+10' is bigger than '+5'.
This is not always true, because if 'x' is large, then (8x + 5) could be bigger than (4x + 10).
Example: Take x = 3, then
$P(A) = 4x + 10 = 4\times3 + 10 = 12 + 10 = 22$
$P(B) = 8x + 5 = 8\times3 + 5 = 24 + 5 = 29.$
So, P(B) is no bigger than P(A).
Coco says that perimeter of B is always bigger than perimeter of A because '8x' is bigger than '4x'.
This also will not be always true. For small values of 'x', we can have (8x+5) smaller than (4x+10).
Example:- If $x = 0.5 = \frac{1}{2}$, then
$P(A) = 4x+ 10 = 4*\frac{1}{2} + 10 = 2 + 10 = 12$

P(B) = 8x+5 = 8*$\frac{1}{2}$ + 5 = 4 + 5 = 9.

Clearly P(B) is smaller than P(A).

The two perimeter P(A) and P(B) can be equal for a special value of x. This is obtained by equaling P(A) and P(B).

= 8x+5 = 4x+10

Rearranging, we get

8x-4x = 10-5

4x = 5

X = 5÷4 = 1.25

So when x = 1.25, then P(B) = P(A)

If x <1.25, P(B)<P(A)

If x>1.25, then P(B)>P(A).

So neither Iga nor coco is always correct. It depends on the value of x.

Problem: A rectangle has length 4x and breadth 3x. Write a simplified expression for its perimeter.

(a) A small square of side length ' x ' is removed from one corner. Find the perimeter of the square.

(b) Calculate the perimeter of the remaining shape. What do you observe?

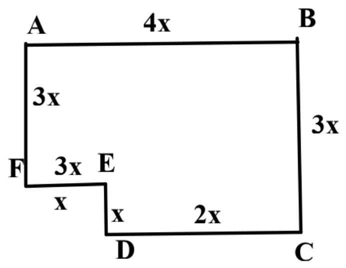
Solution: We have a rectangle of length 4x and breadth 3x.

Perimeter of rectangle = 2(length + breadth)

= 2(4x+3x)

= 2(7x) = 14x

a. Perimeter of small square = Sum of all four sides = 4*x = 4x.
b. Perimeter of remaining figure



= AB + BC + CD + DE + EF

= 4x + 3x + 2x + x + x + 3x = 14x

We observe that the perimeter of the remaining shape is equal to the perimeter of the original rectangle.

A small square is removed from a rectangle. But perimeter is same, only area will be reduced.

Problem: A toy bridge set has eight rods of length.

Eight rods:
x+3, x+3, x+4, x+6, x+6, 2x+2, 2x+4.,3x + 4
Seven rods out of these eight rods are to be arranged to form an equilateral triangle.
Suggest which seven rods will be used and how they will be arranged to make the triangle?
Write an expression for the length of each side of the triangle?

Solution: We are given 8 rods and we are supposed to use only 7 rods to create an equilateral triangle. First we will figure out which rod to leave out.
Since we want to make an equilateral triangle
(all 3 sides are equal), using 7 rods, so definitely the sum of length of these seven rods must be divisible by 3.
Sum of all 8 rods = (x+3) + (x+3) + (x+4) + (x+6) + (x+6) + (2x+2) + (2x+4) + (2x+4)
= 11x + 32.
Now we will leave out different rods and find out if the sum of remaining 7 rods is divisible by 3.
Leave out (x+3)
Sum of 7 rods = (11x+32) - (x+3) = 10x + 29
Not divisible by 3.
Leave out (x+4)
Sum of 7 rods = (11x + 32) - (x +4)
= 10x+28
Not divisible by 3.
Now leave out (x+5),
Sum of 7 rods = (11x+32) - (x+5) = 10x+27
Not divisible by 3.
Leave out (2x+2)
Sum of 7 rods = (11x+32) - (2x+2)
= 9x+30
This exactly divisible by 3.
We have (9x+30)÷3 = 3x+10
It means each side of the equilateral triangle would be (3x+10)
So now we have 7 rods [except(2x+2)] and we have to arrange them in such a way that their sum is (3x+10).
So side 1 can be (x+6) + (2x+4) = 3x+10
Side 2 can be (x+6) + (2x+4) = 3x+10

Side 3 cari be (x+3)+(x+3)+(x+4) = 3x+10

So we have to remove the (2x+2) rod and use the 7 rods in above manner to make an equilateral triangle.

Problem: A toy construction set has 7 rods of length given by x + 1, x + 5, x + 2, x+ 4, x+ 3, x+ 3, 2x + 1.

Six rods out of these seven rods are to be arranged to form an equilateral triangle.

Can you figure out which six rods are to be used? Also show the expression for each side of the triangle?

Solution: We have 7 rods of length. Their combined total length is

Total length = (x+1)+(x+5)+(x+2)+(x+4)+(x+3)(x+3)+(2x+1)

= 8x+19.

All the sides of an equilateral triangle are equal in length. So the perimeter of a equilateral triangle should always be divisible by 3. So we leave out (2x+1)side, the sum of other six sides would be (8x+19)-(2x+1) = 6x+18.

Which is divisible by 3. So the perimeter of the equilateral triangle is 6x+18 and length of each side is 2x+6.

Now we have to arrange these six rods (except(2x+1)) in three groups so that their sum is 2x+6.

A combination can be

Side 1:- sum of (x+1)+(x+5) = 2x+6

Side 2:- sum of (x+2)+(x+4) = (2x+6)

Side 3:- sum of (x+3)+(x+3 = (2x+6)

So our equilateral triangle has each side of length (2x+6) and total perimeter of 3(2x+6) = 6x + 18.

Type : Time Zone Conversions

Problem: The time in London is 5 hours behind the time in Paris. A flight leaves London at 9:20 AM for Paris. The flight takes 7 hours.

a). Write a formula in words linking the time in London and in Paris?

b). Calculate the time in Paris when the flight arrives?

c). What will be the local time in London when the flight reaches Paris?

Solution: We are given that time in London is 5 hours behind (-) the time in Paris.

London Time = Paris Time - 5 hours.

or L = P - 5hrs.

Or we can write,

P = L + 5hrs

Paris time = London time + 5hours
b) Flight leaves London at 9:20 AM and duration of flight is 7 hrs.
So Local time in London when flight reaches
Paris = 9:20 AM + 7 hrs.
= (9:20 + 3hrs) + 4 hrs.
= 12:20 PM + 4 hrs.
= 4:20 PM.
So the time in London (Local time) when flight reaches Paris is 4:20 PM.
c) We know time in Paris 5 hours ahead of London.
So time in Paris = London time + 5 hrs.
= 4:20 PM + 5 hrs.
= 9:20 PM.
So the flight would arrive in Paris when the time in London is 4:20 PM and time in Paris is 9:20 PM.

Problem: Lucy boards a train from Rome at 2:30 pm to travel to Venice. The train ride takes 8 hours. Venice is 5 hours ahead of Rome time.
(a) Write a formula in words linking the time in Rome and Venice?
(b) What will be the time in Rome when Lucy arrives in Venice?
(c) What would be the local time in Venice when Lucy reaches there?

Solution: (a) Venice is 5 hours ahead of Rome.
So
Time in Venice = Rome time + 5 hours
Or in reverse we can write,
Time in Rome = venice time - 5 hours.
(b) Lucky boards the train in Rome at 2:30pm & the journey took 8 hours.
So time in Rome when she reaches
= 2:30pm + 8 hrs
= 10:30pm.
(c) Venice is 5 hours aheads of Rome. so if the time in Rome is 10:30pm, the time in venice would be
10:30 pm + 5 hrs
= (10:30pm + 2hrs) + 3hrs
= 12:30am + 3hrs
= 3:30am

Rules for Solving Time Zone Questions

Time zone questions generally tells you about the time difference between two places. A person leaves place A and has to reach place B. The duration of trip is given and you will be asked to find the local time in place A and B when the person reaches place B.

1. Note the time difference between the two places.
   Pay attention to which place is ahead or behind the other in terms of time.
2. If person starts from A, they add the journey time to the departure time to get the (A) arrival time (B) local time (in A) of arrival in B.
   For example: A person leaves place A at 2 pm and travels for 3 hours to reach place B.
   So the local time in A when the person would reach B would be
   Local time in A = Departure time + Journey time
   = 2pm + 3hrs
   = 5pm
3. To find the destinating local time, use the formula in part (a).
   If the destination is ahead \implies add the time difference to answer of (b).
   If the destination is behind \implies subtract the time difference from to answer of (b).

Problem: A box of pencils contain 15 pencils. Write a formula in words and symbols linking the pack and number of pencils.
Use your formula to find the number of pencils in 5 boxes. Also calculate the number of boxes needed for 120 pencils?

Solution: A single box contains 15 pencils.
So in words, we can write
Number of pencils (P) = Number of Box(B)*15
= P = B*15
P = 15B
Number of pencils (P) in 5 Boxes (B) would be
P = 15*B
= 15*5
= 75pencils
If there are 120 pencils (p), then the number of box (b) would be
= 15B
Dividing both sides by 15, we get

$\frac{P}{15} = B$

Or $B = \frac{P}{15}$

Now P = 120 pencils

$B = \frac{120}{15} = 8$ boxes

Hence 8 boxes are needed for 120 pencils

Problem: Rafael is 4 years older than Novak. If Rafael's age is R and Noval's age is N, write a formula linking their ages?

a. Find Rafael's age if Novak is 18 years old?
b. How old will each be in 10 yrs time?
c. Will the formula be valid in 10 yrs time? Explain?

Solution: It is given that Rafael (R) is 4 years older than Novak (N).

So R = N + 4

Or we can also write

N = R - 4

a. If Noval (N) is 18 years old than Rafael's age (R) would be
   R = N + 4
   = 18 + 4 = 22 years
b. In 10 yrs times,
   Rafael's age would be (R + 10)
   = 22 + 10 = 32 years
   Novak's age would be (N + 10)
   = 18 + 10 = 28 years
c. The formula, R = N + 4 is still valid after 10 years because age difference remains same over time.
   It means 32 = 28 + 4
   Which is true.

Problem: A mother is four times as old as her daughter .

Let mother's age = M and daughters age

= D (a) Write a formula linking their ages?

(b) Find the mother age if the daughter is 12

Will the same formula be valid in 12 years time? If not, suggest what the formula would be in 12 years time?

Solution: The matter is four times (4*) as old as her daughter.

a. In words, we can write
   Mother's age = Three time Daughter age
   M = 4*D
   M = 4d
b. If the Daughter's age (D) is 12 years, then using the above formula,
   M = 4*D = 4*12 = 48 years
   The mother's age is 48 years

c. In 12 years time, the Mother's age will be 48 + 12 = 60 years and Daughter age will be 12 + 12 = 24 years

If we use the formula,

M = 4D and use the new age we will se 60 ≠ 4*24 (4*24 = 96)

So we can see that the same formula is not valid in 12 years time.

Reason: The mother-daughter age ratio will change with time.

The mother's age is 60 years and daughter's age is 24 years. A new formula could be

60 = 24 + 36

Mother's age = Daughter age + 36

M = D + 36

Problem: Alex is twice as old as his sister Dora. If Alex's age is A and Dora's age is D.

a. Write a formula in words and symbols linking their age.
b. If Dora;s age is 5 yrs, calculate Alex's age.
c. How old will each of them be in 10 years time.
d. Is the formula you obtained in (a) part will still be valid? If not, suggest a formula that will be valid in 10 years time?

Solution: We are given that

Alex (A) is twice as old as his sister Dora(D)

a. In words, this becomes
   Alex age = Twice the Dora's age
   = 2 times Dora's age
   In symbol, this beocmes
   A = 2*D
   A = 2D
b. If Dora's age (D) is 5yrs, then using the above formula,
   A = 2*5 = 10 years
   So Alex age would be 10 years if Dora's age is 5 years.
c. In 10 years time, their age would be present age plus 10 years
   Alex's age = 10+10 = 20 years
   Dora's age = 5+10 = 15 years
d. The formula in part (a) is
   A = 2D
   If we put their new age in the formula, we see that
   20 ≠ 2*15 (2*15=30)
   So we see that the formula is not valid.

New formula :- Alex age can be written as

20 = 15 + 5

Alex's age = Dora's age + 5 years

A = D + 5

Problem: The time T(in minutes) needed to roast a chicken is given by

T = 30

Where W is the weight of the chicken in kilograms. Using the above formula, calculate

a. Time needed to roast a 2.5 kg chicken?
b. The weight of chicken if it takes 135 minutes to roast?

Solution: The time T(in minutes) to roast W(kilogram) of chicken is given by

T = 30W

Or W = T ÷ 30 = T/30

a. If w = 2.5kg, then time needed is
   T = 30*25 = 75 minutes
b. If the time is 135 minutes, then putting T = 135 in the formula, we get
   135 = 30W
   Dividing both sides by 30, we get
   W = 135÷30 = 4.5 kilogram.
   So the weight (in kg) of chicken will be 4.5kg.

Problem: The time T (in minutes) needed to fill a car fuel tank given by

T = 5L + 10

Where L is the number of litres of fuel using the above formula, calculate:-

a. How long to fill in 10 litres of fuel?
b. How many litres of fuel can be filled in 50 minutes?

Solution: The time T(minutes) needed to fill L litres of fuel in a car is given by

T = 5L + 10

a. If fuel volume is L = 10litres, then
   T = 5*10 +10 = 50 + 10 = 60minutes
   So it will take 60 minutes (1 hour) to fill 10 litres of fuel.
b. If the time (T) is 50 minutes, then the amount of fuel (L) would be
   T = 5L + 10
   50 = 5L + 10
   Subtracting 10 from both sides,
   40 = 5L
   Dividing both sides by5, we get L = 8 litres

So 8 litres of fuel will be filled in 50 minutes.

Problem: In an electric circuit, the current (I) and voltage (V) are related as.
V = 50I
Where V is the voltage in volts and I is the current in Ampere.
Use this formula, to calculate

a. Amount of current if voltage (V) is 220 volts
b. If the current is 2.5 Ampere, calculate the voltage applied?

Solution: The formula is V = 50I
Where V is the voltage (Volts) and I is the current (in Ampere).

a. If voltage (V) = 220V, then the current would be
   220 = 50*I
   Dividing both sides by 50, we get
   I = 220÷50 = $\frac{220}{50}$ = 4.4 Amp
   So the current in the circuit is 4.4 Amperes.
b. If the current (I) is 2.5 Amperes, the voltage in the circuit would be
   V = 50*I
   = 50*2.5
   = 125 volts

Problem: A school celebrated teacher's day by distributing fountain pens to its students. The school used the formula
P = 25 + 10
Where P is the number of pens and S is the number of students. Use this formula to :-

a. To calculate number of pens needed for 50 students.
b. How many students can be gifted if the school has 50 pens.
c. Why do you think extra 10 is added in the formula.

Solution: We are given the formula,
P = 25 + 10
Where P is the number of pens and 5 is the number of students.

a. If number of students (5) = 50
   Then P = 2*50+10 = 100+10 = 110pens
   So 110 pens would be needed to distribute to 5c students.
b. If the number of pens (P) = 50, then
   50 = 2*5 +10
   Subtracting 10 from both sides, we get
   40=25
   Dividing both sides by 2, we get
   S = 20

So 20 students can be gifted if we have 50 fountain pens.

c. The extra 10 is added in the formula as spares/to replace damaged or defective pens.

Problem: A taxi company changes $5 per kilometer of ride plus a fixed change of $4. Using your own letters for this question, write a formula in words and symbol linking the tool change with the distance travelled.

Solution: We choose letter in such a way it becomes easy to know their meaning.

Let us define C to represent total change (cost) of travel.

D = Distance travelled.

Then we are given that the taxi company changes $5 per kilometer.

So changes if D kilometer are covered would be 5D.

A fixed change of $4 is added no matter what the distance is covered.

So final change becomes

$C = 5D + 4$

In words this becomes

Total change = (5 times distance covered in km)

+ $ 4

Problem: A printing shop changes $0.10 per page plus a fixed change of $1.00. Choosing your own letters, write a formula connecting the total cost with the number of pages printed.

Solution: We are given that

Printing shop changes $0.10 per page so cost becomes $0.10*Number of pages.

There is a fixed cost of $1.00. So total cost becomes

Total cost = $0.10*Number of pages + $1.00

Using symbols like c - Total cost of printing in dollars.

N = Number of pages.

The above formula becomes.

$C = 0.10N + 1.00$

Or $C = 0.1N + 1$

Problem: A swimming pool changes a fixed change of $6.00 per entry and $3.00 per hour spent in the pool.

Write a formula in words and symbols linking the total change with the number of hours spent inside the pool.

Solution: Let us call

C = Total changes of swimming pool in dollars

H = Number of hours spent in the pool.

Then the formula becomes

C = 3H + 6

Or Total change = 3 times number of hours in pool plus six.

In mathematics, we use signed numbers to show direction, gian/loss or rise or fall in temperature.

Signed numbers are also known as "Number with directions" or "Directed number".

Signed number are used in following ways:-

a. In temperature, they can be used to describe above (+) freezing boiling point and below(-) freezing /boiling point.
b. In elevation, we represent above sea level by (+) and below sea level by (-).
c. In finance, profit is (+) and loss is (-).

Problem: Use signed numbers to describe the following qualities:-

a. 1 second before the start of the race.
b. 2 second after the race.
c. A win of $50.
d. A loss of $20.
e. A ball thrown upward at a distance of 50m above sea level.
f. A submarine moving at 70m below sea level.

Solution: a. -1

b. +2

c. + $50

d. - $20

e. + 50m

f. -70m

Problem: The temperature in a city in the morning was 10°c. In the afternoon, it became 5 degree warmer. Calculate the temperature in the afternoon?

Solution: Temperature in morning = 10° c

In afternoon = 5°c warmer

= + 5°c

So final temperature = 10+5 = 15°c

b. In the right, it becomes 12°c cooler. Find the final temperature.

Solution: Original temperature = 10°c

At night = 12°c cooler = -12°c

So final temperature = +10°c - 12°c = -2°c